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Preface

This thesis is a MSc diploma work by Ke Liu, who studies Geomatics at the University of Gävle (Högskolan i Gävle, HiG), Sweden. This study is performed for and conducted by Lantmäteriet – the Swedish mapping, cadastral and land registration authority. Lantmäteriet provides the data and software, on which this study is based.

Martin Lidberg at Lantmäteriet and Stig-Göran Mårtensson at the University of Gävle offered me this precious chance to undertake such study, provided crucial and kindly help. Tina Kempe calculated another table of statistics separately, helped a lot with patient on the method. This study could not have been accomplished without their kindness, patient and professional guidance. I sincerely express my gratitude to Martin, Stig-Göran, Tina, as well as everybody who cared and contributed to this study.

Abstract

In this study, the connection of a local levelling network to the national height system in Sweden, RH 2000, with GNSS-techniques is investigated. The SWEN 08 is applied as geoid model. Essentially, the method is precise normal height determination with GNSS. The accuracy, repeatability and the affecting elements are tested. According to the statistics, the proposed method achieves 1-cm accuracy level. Suggestions on the general methodology and settings of several elements are proposed based on the statistics for the future application.

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A study of the possibility to connect local levelling networks to the Swedish height system RH 2000 using GNSS

1 Introduction

RH 2000 is the new national height system of Sweden and is thought as the best Swedish height system for the time being (Lantmäteriet, 2009a). It is based on levelling data collected during 25 years form 1979 to 2003 (Lilje, 2006) and realized at some 50 000 benchmarks around Sweden (Lantmäteriet, 2009a). In spite of the high density of benchmarks in most part of Sweden, the availability to the network is far from ideal in some remote regions (See Figure. 1). It is thus thought necessary to occasionally add new control points and improve availability to the network. In some of those places, local levelling networks are available and well established with good internal accuracy. Thus, they could be connected to the national height system RH 2000 by determining the heights in RH 2000 of some well-distributed benchmarks in the local network and perform a one-dimensional transformation. Comparing with conventional method (i.e. levelling), Global Navigation Satellite Systems (GNSS), notably the Global Positioning System (GPS), are thought more efficient (eg. Yang et al., 1999; Featherstone, 2008). However, the feasibility and accuracy of GNSS height determination, which aimed at connecting levelling control networks using SWEN 08 as geoid model needs to be investigated further.

The GNSS-derived heights are the ellipsoidal heights referred to the surface of the GRS80 ellipsoid while the physically meaningful height, the orthometric height or normal height, referred to the geoid or quasi-geoid. Their relation can be expressed as Figure 2 and by Equation (1) simply.

$$H=h-N \tag{1}$$

where H is the normal height, h is the ellipsoidal height and N is the geoid height. This equation demonstrates the possibility of GNSS levelling: h is measured with GNSS, thus once N is known, the normal height H can be calculated. Note that theoretically, the plumb line does not always coincide with the normal of the ellipsoid, as shown in Figure 2, but this inaccuracy is so small that it can be omitted in almost all applications (Hofmann-Wellenhof, 1997; Mårtensson, 2002).

Figure 1. *The extent of the third precise levelling network of Sweden (Lantmäteriet, 2009a)*

For Sweden, the (quasi-)geoid model SWEN 08 is the latest and the most accurate geoid model (Ågren, 2009). It has two versions: the one denoted as "SWEN 08_RH 2000" is adapted to the height system RH 2000 and the other version, named "SWEN 08_RH 70 is adapted to the old height system RH 70. In other words, they are essentially the same but adapted to the different height systems. RH 2000 is the only one discussed here, so "SWEN 08_RH 2000" is referred to as "SWEN 08" for short in this report. SWEN 08 inherits the Swedish gravimetric geoid model KTH 08 and is further improved by fitting to "a large number of geometrically determined geoid heights" (Ågren, 2009) whose residual had been modelled considering postglacial land uplift and applying a smooth residual surface (Lantmäteriet, 2009b; Ågren, 2009). Therefore, it is the optimal geoid model available for the time being with good accuracy; the standard error is 10-15 mm in Swedish mainland except for a small area in the northwest which is hardly covered by the third precise levelling (See Figure 1). The standard error in the geoid model in that area is estimated to around 5-10 cm (Lantmäteriet, 2009b; Ågren, 2009). In this study, SWEN 08 is applied not only because of the rule that "the latest published version should be used" (Lantmäteriet, 2009b) but also because of its expected excellent accuracy.

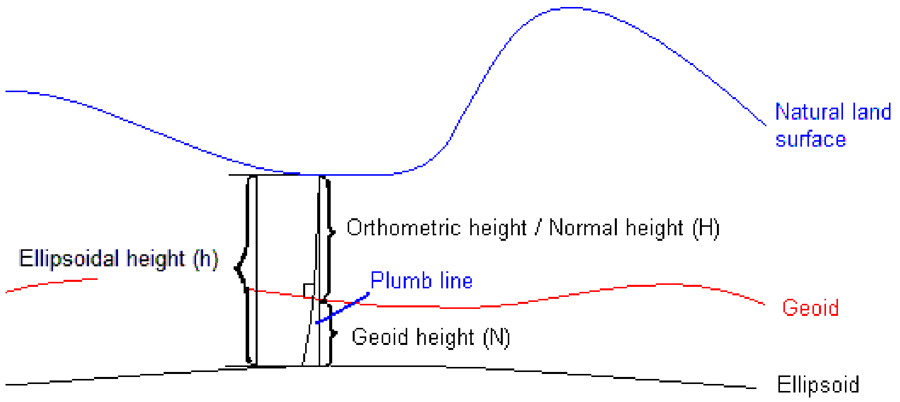

Figure 2. *The relation between height above the ellipsoid, normal height, and the (quasi-) geoid*

1.1 Review on former studies

Since the early days of geodetic applications of GNSS technology, the idea of height determination has been proposed and tested. The National Geodetic Survey of the U.S. (NGS) investigated control survey projects with GPS in early 1983, showed that GPS survey "meet a wide range of engineering requirement in vertical control" (Zilkoski, 1990). Up to recently, the difference in reference surfaces between GNSS determined ellipsoidal height and physically meaningful normal height has been thought as the major problem (eg. Engelis, 1984; Zilkosik, 1990; Featherstone, 2008). The overall methodology was systematically proposed and tested by Engles (1984, 1985), generally following the Equation (1) to convert the ellipsoidal height to normal/orthometric height. Such method is called GPS-levelling method (eg. Zilkoski, 1990) or coincide fitting method (eg. Hu et al., 2004) in relevant study.

In the “GPS-levelling” method, the geoid height turns to be the crucial part affecting the accuracy of resulting normal height, since the GNSS-determined ellipsoidal height have relatively high accuracy (Yang et al., 1999; Mårtensson, 2002 and Benahmed Daho et al., 2006). Regarding the acquirement of the critical geoid height, Yang et al. (1999), Mårtensson (2002) and Featherstone (2008) concluded that in a wide range of applications, provided that the area is small and/or the surface of geoid is flat, the geometric method without gravity correction is thought accurate enough. I.e., include benchmark with known normal height in the network of GNSS survey, calculate the geoid height in such positions, and use methods of interpolation to determine the geoid height of any other location. This method has been proved in many studies, for example, Becker et al. (2002) and Mårtensson (2002). Mårtensson (2002) achieved the relative accuracy of ±10 mm per 10 km using the geometric geoid model. However, when the study area is much larger and the surface of geoid is not flat, some corrections are thought necessary (Engelis et al., 1985; Yang et al., 1999 and Benahmed Daho et al., 2006). Yang et al. (1999) studied the accuracy and contributing error sources of a geoid model obtained with geometric method in a relatively small area (Hong Kong), proposed that incorporation of a geopotential model and a digital terrain model can dramatically improve the accuracy. An accuracy of 2 - 3 cm was achieved in Hong Kong.

1.2 Aim and objectives

However, in the former studies summarized above, the authors made their own geoid model mainly because no other accurate geoid model was thought available. It is obvious that the accuracy of the resulting geoid model differs and the result of normal heights is seriously affected with these uncertainties of geoid model. Featherstone (2008) argued "the ellipsoidal height is inherently less accurate than horizontal position" due to the various errors in GNSS measurement. The transformation from ellipsoidal height to normal height worsens the accuracy due to errors of the geoid model applied. Therefore, provided that an accurate geoid model, e.g. SWEN 08_RH2000, is available, it will be interesting to investigate how the accuracy can be improved comparing to the former studies.

The objective of this study is to investigate the possibility for connecting local levelling networks to RH 2000 using GNSS technology. This is in principle determination of normal heights using GNSS, and applying geoid correction using the SWEN 08 geoid model. There is still lack of evidence showing how accurate GNSS levelling might be and what kind of application it is qualified to when a good geoid model like SWEN 08 is available.

2 The GNSS field experiment

This study is based on a GNSS field experiment performed by Lantmäteriet in 2008, which is primarily aimed at establishing a test data set for evaluating the accuracy of GNSS levelling.

2.1 The choice of study area

An area in the north-east of Uppsala, Sweden, around a small village named Gåvsta was chosen for the GNSS field experiment, on which this study is based. A local levelling network exists in Gåvsta, encircled by a loop that consists of benchmarks of the national levelling network in RH 2000. See Figure 3 and 4. Previously, the local network has been connected to the national height system by motorized levelling as a densification of the national network (Becker, 1985). In this GNSS field experiment, some well-distributed benchmarks, in both the national and the local network, were chosen to be re-measured with GNSS in order to establish a test-dataset. Because both GPS-only and GPS/GLONASS receivers are used in the measurement, only the GPS signal has been used in this study. Therefore, the term “GPS” is used on the specific data involved in this study, and the term “GNSS” is used to describe the GNSS data that might be used in this general methodology. Moreover, in this report, sites 1001, 1002, 1003 etc. are referred as sites of ”1000-series” for short. Similarly, sites of “2000-series” refer to sites 2001, 2002, 2003, etc.

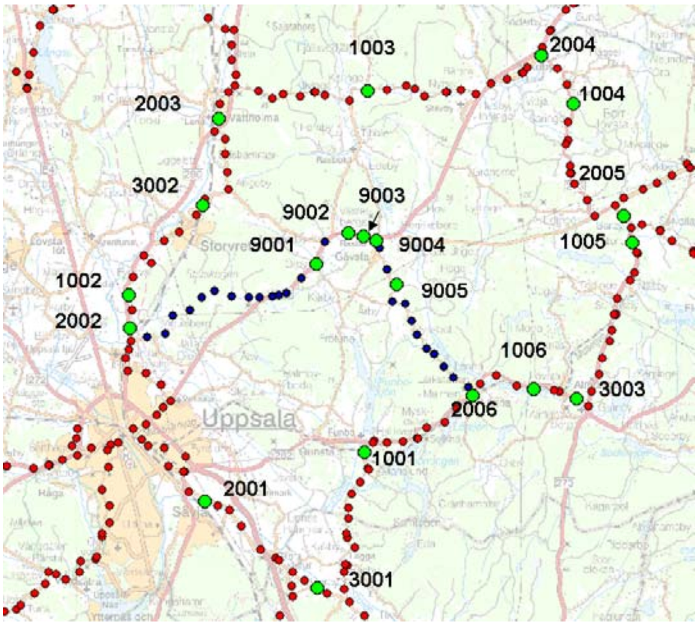

Figure 3. *Sites measured on February 18 – 20, 2008 : the red dots shows the benchmarks of national network, the blue ones shows the sites of densification and the labelled green ones shows the dots re-measured with GPS (Eriksson, 2009)*

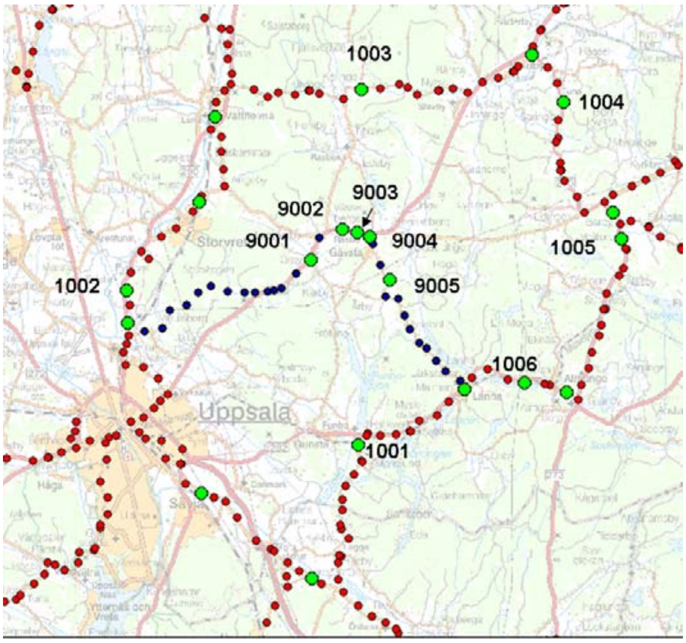

Figure 4. *Sites measured on March 17 – 21, 2008: the red dots shows the benchmarks of national network, the blue ones shows the sites of densification and the labelled green ones shows the re-measured with GPS (Eriksson, 2009)*

The lengths of the GPS baselines are calculated from the approximate horizontal location of each measured site and listed in Table 1. The average length is 16 km. Previous experiences on local control networks using GNSS are usually based on smaller networks with shorter baselines. According to the guidelines for GPS measurements of the Swedish series of handbooks in surveying and mapping "HMK Geodesi GPS" (Lantmäteriet, 1996), baselines are required to be shorter than 10 km when using the GPS L1 frequency only to ensure required accuracy (Lantmäteriet, 1996). However, a levelling loop of RH 2000 is about 100 km. The distance between benchmarks within the levelling lines are about 1 km. But since the diameter of the loops are some 20-30 km, some of the GNSS baselines will have this length while performing a GNSS based densification of the levelling network. Thus, some baselines will exceed the 10 km limit while connecting a local levelling network to RH 2000 using GNSS. Nevertheless, it is thought feasible to keep baselines of this length in this study, because: firstly, "HMK Geodesi GPS" (Lantmäteriet, 1996) was composed 14 years ago based on equipments at that time. With the cancellation of Selective Availability and the improvement on antennas and receivers, the accuracy of GNSS measurement is significant improved. Secondly, it will be tested to use the ionospheric-free linear combination, denoted as Lc, in the analysis. In LC, the effect due to different ionospheric condition in large distances is reduced. Therefore, longer baselines are kept in this study.

***Table 1.** Length of baselines (km)*

1001	1001																		
1002	14.9	1002																	
1003	19.0	16.5	1003																
1004	21.4	25.4	10.9	1004															
1005	17.9	26.6	16.1	7.9	1005														
1006	9.5	21.8	18.0	15.2	9.4	1006													
2001	8.8	11.5	23.2	28.6	26.4	18.3	2001												
2002	13.9	1.8	17.7	26.1	26.8	21.4	9.8	2002											
2003	19.2	10.4	8.0	18.7	22.7	21.8	20.1	12.0	2003										
2004	22.9	25.1	9.3	3.1	11.0	17.6	29.4	25.9	17.3	2004									
2005	18.4	26.3	15.0	6.5	1.4	10.3	26.7	26.6	21.9	9.6	2005								
2006	6.4	18.8	17.0	16.3	11.7	3.2	15.2	18.3	19.8	18.3	12.4	2006							
3001	7.5	18.3	26.3	28.8	24.6	15.4	7.5	16.8	25.2	30.4	25.3	13.0	3110						
3002	15.5	6.1	10.5	20.2	22.7	19.9	15.6	7.5	4.6	19.4	22.1	17.3	21.0	3002					
3003	11.5	24.1	19.6	15.5	8.7	2.3	20.3	23.7	23.9	18.1	9.9	5.5	16.9	22.1	3003				
9001	10.2	10.0	9.5	16.0	16.7	13.1	13.8	10.3	9.3	16.2	16.4	10.7	17.0	6.7	15.4	9001			
9002	11.6	12.0	7.6	13.7	15.0	12.7	16.0	12.5	9.1	13.8	14.5	10.8	18.7	7.8	14.8	2.4	9002		
9003	11.4	12.7	7.7	13.1	14.2	12.0	16.3	13.2	9.8	13.3	13.7	10.2	18.7	8.6	14.1	2.9	0.8	9003	
9004	11.2	13.3	7.8	12.6	13.5	11.4	16.5	13.8	10.5	13.0	13.1	9.7	18.6	9.3	13.4	3.4	1.5	0.7	9004
9005	9.0	14.1	10.3	13.3	12.6	9.0	15.3	14.2	12.8	14.2	12.4	7.1	16.5	11.0	11.1	4.4	3.7	3.1	2.6

Maximum Distance: 30.4 km between Point 2004 and Point 3001
Minimum Distance: 0.7 km between Point 9003 and Point 9004
Average Distance: 15.97 km

2.2 GPS campaigns

The GPS observation, on which this study is based, was performed in the study area in early 2008. Before the GPS survey, a reconnaissance to the study area was undertaken, in order to select benchmarks suitable for GNSS observation, which could be included in this study. They should be easily accessible, open to GNSS signal and, as discussed above, well distributed in the study area (Eriksson, 2010). Eventually, 20 sites are chosen for measurement, as shown in Figure 3.

The GPS survey for the first two days, denoted as Day 1 and Day 2 in this study, was carried out on February 18-20, 2008. Standard commercial antennas of modern generation were used. The same antennas (Leica AX1202GG) are used on all the sites. As shown in Figure 3, 20 sites were measured, including sites of the 1000, 2000, and the 3000-series, which are the benchmarks of the national levelling network, and sites of the 9000-series of the local network. Each site was equipped with a set of receiver and antenna, performing a carrier phase static measurement for 24 hours on each day. The measurement of Day 1 was from 12:00:00, February 18th to 11:59:55, 19th. With 2 hours of re-setup in between, the Day 2 was from 14:00:00, February 19th to 13:59:55, 20th.

The GPS survey for the third and the fourth day, denoted as Day 3 and Day 4, was performed later on March 17 - 21, 2008, following exactly the same procedure as Day 1 and Day 2. However, as shown in Figure 4, only 11 sites were measured, including sites of the 1000-

series of the national network, and sites of the 9000-series of the local network. Ashtech and Javad versions of the Dorne Margolin Type T model antennas were used in Day 3 and Day 4. Three variations of Ashtech models were used separately on the sites of 1004, 9001, 9002 and 9004, and Javad JNSCR_C-146-22-1 antennas were used on all the other sites. See Table 2.

Table 2. *The antenna used in Day 3 and Day 4*

Point Number	Antenna used in Day 3 and Day 4
1001	Javad Positioning System JNSCR_C-146-22-1
1002	Javad Positioning System JNSCR_C-146-22-1
1003	Javad Positioning System JNSCR_C-146-22-1
1004	Ashtech ASH 701941.B
1005	Javad Positioning System JNSCR_C-146-22-1
1006	Javad Positioning System JNSCR_C-146-22-1
9001	Ashtech ASH 700936 E
9002	Ashtech ASH 700936 E
9003	Javad Positioning System JNSCR_C-146-22-1
9004	Ashtech ASH 701945C_M
9005	Javad Positioning System JNSCR_C-146-22-1

The resulting data of measurement for each 24 hours was further split into several sessions of shorter time duration (session length): 1 hour, 24 sessions; 2 hours, 12 sessions; 3 hours, 8 sessions and 6 hours, 4 sessions. The purpose is to simulate measurement with shorter session length and study the impact of session length on accuracy. All the sessions with different session length were saved separately in RINEX format. Besides different session lengths, the complete dataset of this experiment can be used to simulate different circumstances of GPS measurements as required by a specific study, for example, using different antennas and GPS frequency combinations (L1 or Lc, see Chapter 5.3.2), having different degree of freedom etc. It is realized by assigning different options in baseline processing, using only the desired part in this dataset, making some unique combinations out of the original dataset, or adjusting the number of points included in the network etc. In this study, many variations of data and settings in GPS analysis have been tested based on the complete dataset, in order to test the accuracy in different circumstances (see Chapter 4.1).

3 The method of connecting local levelling networks to RH 2000

The purpose of this study is to investigate the possibility to connect local levelling networks to the national height system RH 2000 in Sweden. The methodology applied is generally composed of three parts. Firstly, compute GPS baselines and perform network adjustment in a free network. Secondly, transform this free network into RH 2000 by using a geoid model and a regional fit to known points in RH 2000. Finally adjust the local levelling network to some GPS-determined points in RH 2000 from the second step.

In some more detail, the following method is proposed in this study to connect local levelling networks to RH 2000 (see Figure 5): firstly, free network of GPS measurement is calculated and adjusted. The resulting ellipsoidal heights of the benchmarks in the local network are transformed into approximate normal heights using SWEN08 (Ågren, 2009) as geoid model (Chapter 3.1 and 3.2). Secondly, the resulting network of approximate normal heights is aligned to the known heights in RH 2000 on benchmarks included in the network by applying a one-dimensional 3-parameter vertical transformation (an inclined plane). With this transformation, the GPS obtained free network is adjusted to the network of RH 2000 and the GPS-obtained approximate normal heights of the local network are corrected. An indicator of quality of this GPS-determined network is also calculated in this step. See Chapter 3.3. Thirdly, the local network is aligned to the GPS-obtained network by performing a 1-parameter vertical transformation using the benchmarks in the local network re-measured with GPS as common points. With this transformation, the translation value between the local and the national system are calculated. Thereby, the heights in RH 2000 of the other benchmarks in the local network, which are not re-measured with GPS, are computed. See Chapter 3.4.

The GNSS software Trimble Total Control (Trimble Navigation Ltd., 2002), denoted as “TTC” in this report, have been used in this study for baseline calculation and network adjustment. The Gtrans transformation utility (Lantmäteriet 2009c) was used for coordinates transformation and network fitting, which is essentially a program for coordinates/ heights transformation.

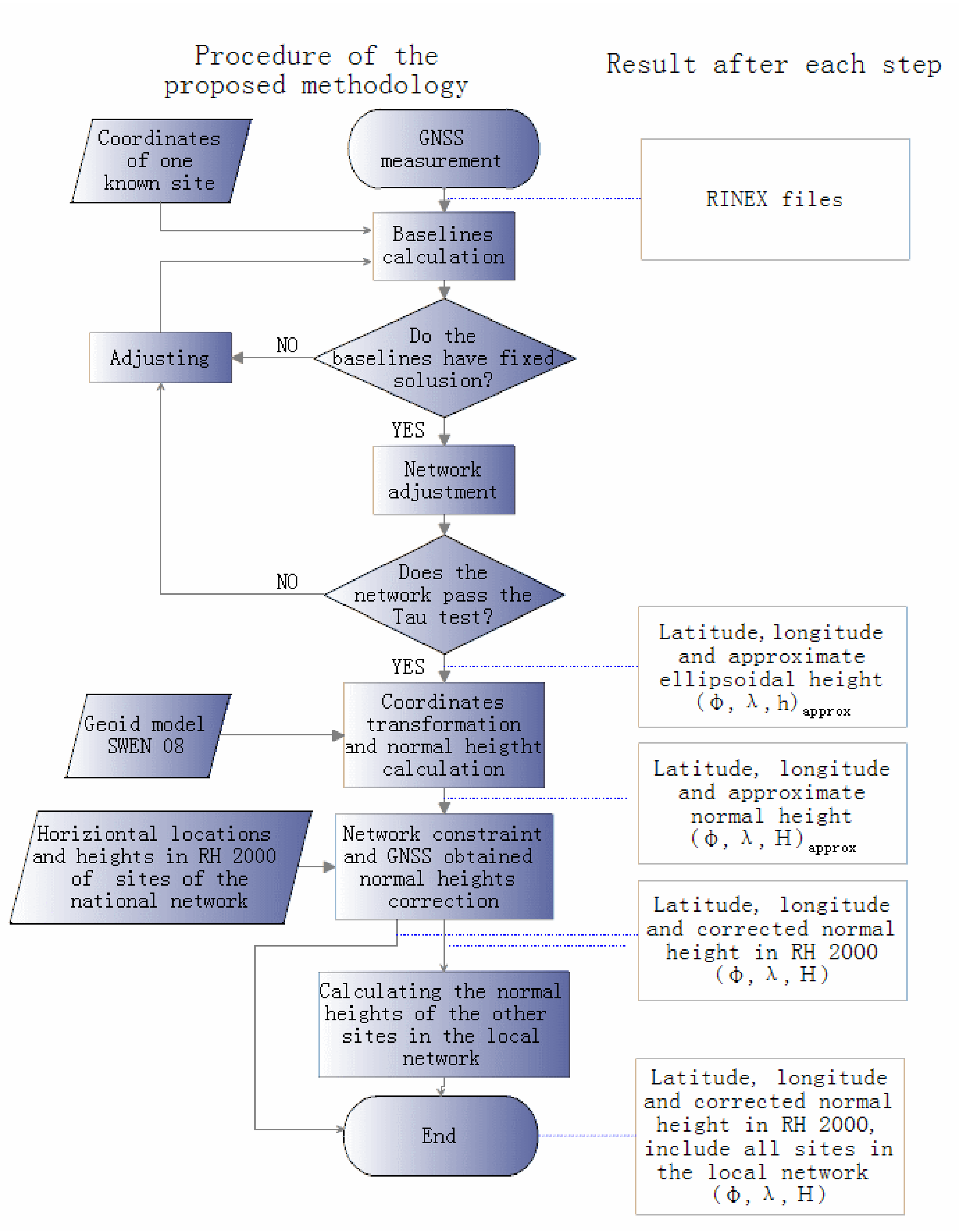

Figure 5. *The flow chart on the method of connecting local levelling networks to RH 2000*

3.1 Baselines processing and network adjustment

GPS measurements are processed with Trimble Total Control (TTC), to calculate the baselines and construct a free network in order to compute the approximate horizontal positions and the ellipsoidal heights. In this first step, 1 point must have a good approximate position known. The options of baselines processing (See Table 3) are almost identical for all the strategies but the wavelength applied (L1 or Lc) might vary. The basic criterion in this step is that all the baselines must have fixed solutions (the phase ambiguities determined to integers). See the step of “Baselines calculation” in Figure 5.

The GPS measurement is constructed as a free network mainly based on the theory that the network of GNSS measurements have relatively good internal accuracy, but it might be tilted and translated due to the errors in GNSS measurement and in the geoid model. Therefore, it is thought to be a better option to construct a free network with good internal accuracy without any interference of external errors, ensure the internal accuracy with free network adjustment, and then fit it to the network of known heights to absorb such tilt, constrain the GNSS-derived network and correct the GNSS-derived heights. To construct such free network, it is necessary to have one point fixed in the network because GPS baselines themselves contain references of scale and orientation, only one reference of location (known sites) is needed for the adjustment (Zhang et al., 2005). In this study, point 1001 is assigned to be the fixed point with known Cartesian coordinates in most computations and point 2001 is also tested as the fixed point in some trials.

***Table* 3.** *Baselines Processing Options*

Tab in TTC	Options in TTC	Value
Parameter	GPS Cutoff	10° by default, may be increased if needed
	Preference	Prefer P code
	Frequency	Mainly L1 Only, Lc Only is tested for Day 1
	Orbit Type	Precise (IGS Final Orbits)
	Processing Interval	15 s or 5 s, Forced Interval: Yes
Filter	Use Following Solutions	Fixed/L1, Fixed/Lc
GLN Sats	Disable the GLONASS	Disable All (Use GPS satellites only)

Note: TTC default values of other options remain.

The 3D free network adjustment was then performed with the algorithm of least square adjustment, aimed at evaluating and ensuring the internal accuracy of the network, detecting potential distinct systematic errors and gross errors (Hofmann-Wellenhof et al., 1997). According to the procedure of TTC, such adjustment can be realized with “free network” adjustment, and then there is an option to also perform a “biased” adjustment. The former operates without any reference point; while the latter introduce the control points with known horizontal and/or vertical positions as known, i.e. the fixed points. The Cartesian coordinates of each point is updated and the quality of the network is evaluated in the network adjustment. In this study, the network adjustment has been performed as “biased” adjustment using only one point as fixed. The result after network adjustment is a network where the internal accuracy of the network is determined by the GPS observations, but it is not disturbed by constraints from known points. But it might be tilted, rotated and translated with respect to the correct positions because it has not been constrained to more than one point. So, the GPS-obtained ellipsoidal heights here are an approximation that needs to be further corrected.

The resulting (approximate) 3-dimentional SWEREF 99 Cartesian coordinates of each point are output in the K-file format. This format is essentially a text file with coordinates, developed by Lantmäteriet and the specific format used in the coordinate transformation tool, Gtrans.

Moreover, in Trimble Total Control, an antenna model provides phase centre eccentricity and elevation dependent variation information of a calibrated antenna (Trimble Navigation Ltd., 2002). Normally, if the required antenna model is included in TTC, the baselines processing can be performed without extra preparation. However, in this study, the phase centre variation (PCV) models of Leica AX1202GG and Javad JNSCR_C146-22-1 antennas are not included. Therefore, they must be installed manually before baselines processing. In this study, the phase centre variation model of Leica AX1202GG and the model of the Javad antenna is available on the website of National Geodetic Survey of the U.S. (NGS)[1]. The table of elevation- and/or azimuth-dependent antenna phase centre offsets is copied from the website and arranged into specified format in TTC (see Appendix 1). See Trimble Navigation Ltd. (2002).

3.2 Coordinate system transformations and normal height calculations

Before computing normal heights with Equation (1), the resulting SWEREF 99 Cartesian coordinates of all the points must be transformed into SWEREF 99 TM (the national Transverse Mercator map projection for Sweden) to fit the coordinates system applied by SWEN 08. Then, SWEN 08 is applied as geoid correction to compute normal heights in RH 2000 of each site (see Figure 5). In this study, they are realized with the software of Gtrans. The results are horizontal coordinates in SWEREF 99 TM and normal heights in RH 2000. The resulting normal heights are still in a free network. Therefore, they are approximations and need to be corrected by constraining the free network to the known heights of bench marks in RH 2000.

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[1] http://www.ngs.noaa.gov/cgi-bin/query_cal_antennas.prl?Model=JNS&Antenna=JNSCR_C146-22-1%20NONE

The horizontal coordinates, which required by SWEN 08 to obtain geoid heights, are also approximations from the free network. Theoretically, potential errors in the horizontal domain might affect the geoid heights and thereby affecting the resulting normal heights. Nevertheless, it is thought insignificant and can be omitted in this project. The approximate horizontal coordinates is applied in this study because horizontal locations of the benchmarks of RH 2000 themselves are inaccurate. It is the situation in Sweden and in almost all the other countries that the benchmarks of a height system are not as accurate in horizontal domain as a triangulation point. Thus, even if the free network was constrained in horizontal domain, their latitudes and longitudes are still relatively inaccurate. However, in this study, such errors are thought so insignificant that can be ignored in this area of Sweden. Empirically, the maximum absolute horizontal deviation in static carrier phase GPS measurement is expected around 10 m. The elevation abnormity (ζ), i.e., the difference of geoid height, is computed to be 0.03 mm per meter by average between these investigated benchmarks. Therefore, the error will be 0.3 mm even if a significant horizontal error existed by 10 m in one site. Obviously, theoretically possible errors due to the inaccuracy of horizontal location are so insignificant that it can be ignored in this study. However, it is not proved in this study that the same accuracy can be achieved as a triangulation point in the western and northern part of Sweden where the elevation abnormity (ζ) might be steep. Moreover, theoretically possible errors in horizontal position in the GPS measurement will also cause tilt in calculated baselines. There is lack of investigation in this study on this error. However, the result shows it is acceptable even if it existed in this study (see Chapter 5.1). One possible solution to such problems is to improve the horizontal accuracy of GPS measurement in horizontal domain, such as connecting the network to permanent GNSS stations (CORS) in the SWEPOS® control network.

3.3 Network constraint and GNSS obtained normal heights correction

After calculating the approximate normal heights, the free network is ready to be aligned (fitted) to the known heights of benchmarks of RH 2000 included in the network. By doing this, the free network is adjusted to and made consistent with the network of RH 2000. E.g. the approximate normal heights of the local network in Gåvsta (points of 9000-series) are being corrected. It might need both vertical shift and rotation about the x- and y-axis to perform such constraint. Therefore, a one-dimensional (vertical) 3-parameter coordinate/height transformation (inclined plane transformation) is applied. See Equation (2):

$$H_i = H_{i0} + C_0 - y_{i0}\Delta_{a1} + x_{i0}\Delta_{a2} - V_h \tag{2}$$

where H_i is normal height in RH 2000, H_{i0} is the GNSS determined approximate normal height, C_0 is the vertical shift between the two height systems, $\Delta\alpha_1$ and$\Delta\alpha_2$ are rotation angles about the x-axis and the y-axis, x_{i0} and y_{i0} are the (possibly approximate) horizontal coordinates. V_h is the residual of height for each specific point. The origin of this rotation is in the geometrical centre of the network. Therefore, x_{i0} and y_{i0} are relative to centre of the network. The horizontal coordinates are required with only low accuracy (Hofmann-Wellenhof et al., 1997). The redundant common points here is important because they "enable a least square adjustment and provide necessary check on the computation of the rotation" (Hofmann-Wellenhof et al., 1997). Meanwhile, the standard error of unit weight (S_0) of this transformation is calculated from the residuals according to the following formula:

$$S_0 = \sqrt{\frac{\sum_{1}^{N_p} V_h{}^2}{N_f}} \tag{3}$$

where S_0 is the standard error unit weight of the transformation. N_p is the number of points included. V_h is the residual of height for each specific point. N_f is the number of redundancies (or degrees of freedom).

$$N_f = nN_p - N_c \tag{4}$$

where N_c is the number of parameters applied in the transformation; n is the number of dimension, i.e., n=3 in 3-dimensional Helmert transformation. So, n=1 in this 1-dimensional transformation:

$$N_f = N_p - N_c \tag{5}$$

where N_f, N_p and N_c is identical as portrayed above in Equation (3) and (4). The standard error of unit weight of the transformation is an indicator about how the two networks agree, which is an indicator of the quality of GPS height measurement and normal height calculation with SWEN 08, See Chapter 4.3.1., where all the indicators are explained.

3.4 Height computation of the other sites in the local network

Practically, not all the benchmarks in the local network must be observed with GNSS. The normal heights of the other benchmarks, which were not observed with GNSS, can be subsequently computed by fitting the local network to the GNSS-determined network of normal heights in RH 2000. Based on the former transformation, the levellings are supposed not to be tilted so and the local system should not be tilted when aligned to RH 2000. A one-dimensional (vertical) transformation is therefore applied. The mathematical expression is:

$$H_t = H_f + C_0 - vH \tag{6}$$

where H_t are the heights after the transformation, i.e. normal heights in RH 2000. H_f are the heights of points in the local system before the transformation. C_0 is the systematic height shift of the local levelling network, and vH is the residuals.

After this step, the normal heights of all the benchmarks in the local network have been resolved and therefore the progress of normal heights determination with GNSS is accomplished. That means the local network has been connected to RH 2000.

In this study, all the transformation portrayed above are performed with Gtrans. Some or all of the benchmarks of RH 2000 re-measured with GPS, e.g. sites of 1000, 2000 and 3000-series, are used as common points. Both the result and the statistics of the transformations are saved for further analysis. See Chapter 4.

4 Evaluation of the proposed method

Following the same approach as depicted in Chapter 3, many sets of independent computations of normal heights are performed separately for test. See Chapter 4.1. After obtained the normal heights with GPS, the proposed methodology is evaluated by comparing the GPS-obtained normal heights with their corresponding pre-determined (chapter 2.1) normal heights in RH 2000. Statistics will be preformed to compute indicators in each test computation. The indicators will be arranged, compared and evaluated (see Chapter 4.3) to conclude proposals for future application (see Chapter 5).

4.1 Design of experiments

The methodology proposed in this study was tested using different data and settings. For example, using measurement of different days obtained with different antennas, using different session duration and GPS frequency combinations (e.g. L1 or Lc), simulating measuring more sites with less GPS receivers, and some other reasonable changes on parameters, as shown in Table 4. The purpose of such experiments is to test the accuracy and repeatability of this approach under different circumstances that might exist in applications. By comparing their accuracy, the affecting elements of the proposed method are identified and analyzed. The ultimate goal for this study is to be able to propose an optimal combination of observation and analysis strategy for these kind of survey work. In this study, a full set of data and settings used in a computation is referred as a "strategy". In other words, a “strategy” refers to certain combination of methodology for GPS measurements, options in baseline processing, and network adjustment and transformations applied in order to derive normal heights in RH 2000. Various strategies are designed to realize tests mentioned above, see Table 4. They are organized in 9 groups, numbered with Roman numerals in Table 4, and named according to their attributes.

***Table 4.** The attempts of different data and settings*

Wave Length	Number of points	Day Duration	D1	D2	D3	D4
L1	20	2 hours	×	×	-	-
		3 hours	×(11) I	×(12)	-	-
		6 hours	×	×	-	-
	11	2 hours	×	×	×	×
		3 hours	×(13) III	×(14)	×(21) II	×(22)
		6 hours	×	×	×	×
	11	3×3 hours	× (11s6r) IV	×	○	○
	9	2×3 hours	× (9s6r) V	×	○	○
	11	3 hours	×(17) VI	×(18)	-	-
Lc	11	3 hours	×(15) VII	×(16)	×(23) VIII	×(24)
	9	2×3 hours	× (9s6rc)IX	×	○	○

Legend:
× Calculated in this study
- No data
☐ Not calculated in this study

Note: The Roman numeral refers to the group of strategies and the Arabic numbers in the brackets refer to the number of strategy (see Chapter "The naming of files" in Appendix).

In different strategies, different benchmarks might be used as common points in transformations described in Chapter 3. Generally, all the benchmarks of the national height system, i.e., points of 1000, 2000 and 3000-series, are used as "common points". However, they might not be all included in some strategies, in which only points of 1000-serie or other points are used as common points. The former situation is denoted as "11 sites". The latter situation will be especially specified.

The tested strategies in this study are listed as follows:

I. D1_D2: data of L1, Day 1 and Day 2, 20 sites, 2 – 6 hours session duration. Standard antennas

II. D3_D4: data of L1, Day 3 and Day 4, 11 sites, 2 – 6 hours session duration. Dorne Margolin choke ring antennas

III. D1_L1_1000: data of L1, Day 1 and Day 2, 11 sites, 2 – 6 hours session duration. Standard antennas

IV. 11s6r: data of L1, Day 1 and Day 2, a simulation of measuring 11 sites with 6 receivers in 3 sessions (see Table 5), 3 hours session duration. Standard antennas

V. 9s6r: data of L1, Day 1 and Day 2, a simulation of measuring 9 sites with 6 receives in 2 sessions. Of these 9 points, 6 are known bench marks in the national levelling network, while 3 are points in the local network. Standard antennas

VI. D1_D2_L1_2000: data of L1, Day 1 and Day 2, 11 sites in which sites of 2000-series are used instead of the 1000-series as known ones, 3 hours session duration. Standard antennas

VII. D1_Lc_1000: data of Lc, Day 1 and Day 2, 11sites, 3 hours session duration. Standard antennas

VIII. D3_Lc: data of Lc, Day 3 and Day 4, 11sites, 3 hours session duration. Dorne Margolin choke ring antennas

IX. 9s6rc: same as 9s6r, but using Lc.

Most strategies listed above are sufficiently understandable without further explanation except “IV - 11s6r”, “V - 9s6r, and IX - 9s6rc”. The strategy IV, “11s6r”, is a simulation of measuring required sites with fewer antennas. In this case, the whole network of 11 sites is measured with 6 receivers and covered in 3 sessions (see Table 5). The session length of each session is 3 hours. There is 1 hour reserved between two sessions for moving equipment. Therefore, practically, the whole network can be measured in 11 hours in a long workday. However, in this study, the time span for moving the equipment is assumed to be 3 hours in order to use the existing GPS measurement of 3-hour session duration from the test data set (see Chapter 2) without further treatment. Thus, provided that the first session (Session A) starts from the first hour, the second session (Session B) should start from the seventh hour, etc. (see Table 6). Moreover, in this study, measurement of each 3-hour in Day 1 and Day 2 was tried as the first session, i.e., Session A, of the simulated measurement. For example, in the first attempt (referred as “Measurement A” in Table 6), Session A begins in the first hour of D1, and then in the next attempt, i.e., Measurement B, it begins in the fourth hour, until the last attempt, in which the Session A starts from the twenty-second hour of Day 2. This “redundant” procedure is designed to exclude the time-dependent interference (the ionosphereric effects and interferences of multi-path reflection etc.). Therefore, there are totally 16 sets of computations in “11s6r”. See Table 6.

Table 5. *The plan of measuring 11s6r*

Session	A	B	C
Points included	1001	1001	1003
	1002	1005	1004
	1003	1006	1005
	9001	9002	9002
	9002	9004	9003
	9005	9005	9004

Table 6. The changing of the beginning of the first session

Measurement	Session A	3 hours in between	Session B	3 hour in between	Session C
A	D1, H1-H3	D1, H4-H6	D1, H7-H9	D1, H10-H12	D1, H13-H15
B	D1, H4-H6	D1, H7-H9	D1, H10-H12	D1, H13-H15	D1, H16-H18
C	D1, H7-H9	D1, H10-H12	D1, H13-H15	D1, H16-H18	D1, H19-H21
...	...	...	...	...	...
P	D2, H22-H24	D1, H1-H3	D1, H4-H6	D1, H7-H9	D1, H10-H12

Note 1: This table is an illustration for "11s6r" therefore 3 sessions are included. For "9s6r" and "9s6rc", the principle is the same but there are only 2 sessions.
Note 2: H1 in this table means the first hour. H1-H3 means the GNSS measurement from the first hour to the third hour.

Similarly, the strategies of "9s6r" and "9s6rc" are simulations of measuring 9 sites with 6 receivers. They follow almost the same procedure as "11s6r" with even fewer sites (see Table 7): 3 sites in the local network and 6 sites in the national network. The measurement needs 2 sessions. With the session length of 3 hours and 1 hour for moving the equipment, the measurement can be finished in 7 hours in one workday. In this study, the time span between two sessions is also assumed to be 3 hours for the same reason explained above in the strategy of "11s6r". The test of "9s6r" and "9s6rc" are also aimed at studying the affection on accuracy of less degree of freedom in both common and "unknown" points (points of the local network). Similarly, there are 16 sets of computations for each wavelength in this strategy, totally 32 sets of computations. The plan of measurement is listed in Table 7.

Table 7. The plan of measuring 9s6r and 9s6rc

Session	A	B
Points included	1001	1003
	1002	1004
	1003	1005
	1005	9001
	1006	9003
	9001	9005

Following all the nine groups of strategies descried above, calculation of the normal heights of sites in the local network, i.e., the connecting, is performed separately for comparison.

4.2 Check with known network

Each methodology applied in this study is evaluated by comparing the GPS-obtained normal heights of bench marks in the local network to their known heights determined by precise levelling. The evaluation is performed with the same one-dimensional transformation following Equation (6), using the GPS-determined heights as "from" system and the pre-determined heights as "to" system. Only the deviations between two systems, i.e. C_0 in Equation (6), are interesting here as an indicator of network error. C_0 of a session is the mean of the deviation on all common points:

$$C_0 = \frac{\sum_{i=1}^{n} dH_i}{n} \tag{7}$$

where n is the number of common points in that session and dH_i is the deviation between the GPS-determined height and the real height in RH 2000 in a certain point. Moreover, the standard error of unit weight for each session is calculated following equation (3) and (5). See Chapter 4.3.1 for the implication of those indicators.

In this study, the procedures above are executed with the transformation software Gtrans.

4.3 Statistics on the accuracy

Following the same procedures as described above from connecting networks (see Chapter 3) to check with known network (see Chapter 4.2), the GPS measurements of each session using each strategy are calculated separately for each day. Each of them yields a complete set of results and statistical measures of the errors. Such redundancies provide data for analysing the accuracy of this methodology and its affecting factors. Indicators of error and precision are computed in this step, on which the subsequent analysis are based. In this study, Microsoft Excel is used for the statistics, and an Excel VBA macro is developed to automate some processes.

Statistical indicators

The statistics focus on two steps individually: firstly, GNSS network adjustment and computation of normal heights including transformation to benchmarks in the national height network of RH 2000 (see Chapter 3.3) and secondly, comparison with known RH 2000 heights of the local network (see Chapter 4.2).

In the first step, residuals of the network fitting (vH in Equation (2)) are considered indicating the inconsistency, i.e. internal errors of the GPS-determined network. The vertical shift C_0 is not considered as "error" because those two networks are not expected to be vertically identical. The linear vertical deviation and tilt (see Chapter 5.3.4 on discussion about linear and non-linear errors) are supposed to be eliminated in this transformation. Therefore, the standard error of unit weight of the transformation, S_0, calculated from vH following

Equation (3), is considered to be the interesting parameter from this step. It is obvious that S_0 is an estimation of how much the GPS-obtained normal heights vary around the true value after the transformation. S_0 is also an indicator of the network fitting. Since the pre-determined network (the national height network of RH 2000) is considered correct, S_0 is thus an indicator of the internal quality of the GPS-obtained free network. The RMS (Root Mean Square) of S_0, RMS_{S0}, of all sessions in a certain day using a certain strategy is calculated to obtain an expected value of S_0 in all the relevant sessions. The calculation of RMS is defined below:

$$RMS = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}} \tag{8}$$

where, when computing RMS_{S0}, x_i is S_{0i} and n is the number of sessions.

In the second step, the purpose is to find out to which uncertainty a local levelling network may be connected to RH 2000 using GNSS and the proposed strategy. In the transformation, both the vertical shift (C_0 in Equation (7)) and the residual on an individual point (vH in Equation (6)) are considered as the "error". The former is the deviation between two networks, i.e. the error of the height level of the connected network. Because theoretically, the two networks are expected identical if no error occurs in this step. The vertical shift C_0 is thus the uniform height deviation between the two networks. The latter, vH, is the error in an individual point besides C_0. Therefore, S_0 is computed as an indicator of error existing in each individual point besides the error of the network. The RMS_{So} is also calculated following Equation (8) to evaluate the expected value of S_0 in all the relevant sessions in a certain day using a certain strategy. Besides, the RMS of C_0, RMS_{C0}, of all sessions in a certain day using a certain strategy is also calculated to evaluate the expected C_0 in that circumstance. RMS_{C0} is a measure of how well a local network can be connected to RH 2000 using GPS and the applied methodology. Furthermore, the maximum and minimum of C_0 (Max(C_0) and Min(C_0)), the difference between maximum and minimum (ΔC_0), the arithmetic mean of C_0 ($\overline{C_0}$) and the standard deviation of C_0 (S_{C0}) of all sessions in a certain day using a certain strategy are all computed for further analysis. Their equations are defined below for clarification. RMS of C_0 (RMS_{C0}) is computing follow equation (8). The values ΔC_0, $\overline{C_0}$, and S_{C0} are computed as follows:

$$\Delta C_0 = C_{0max} - C_{0min} \tag{9}$$

$$\overline{C_0} = \frac{\sum_{i=1}^{n} C_{0i}}{n} \tag{10}$$

where n is the number of values calculated.

$$S_{C_0} = \sqrt{\frac{\sum_{i=1}^{n} (C_{0i} - \overline{C_0})^2}{n-1}} \tag{11}$$

As mentioned above, RMS_{C0}, a measure of the expected deviation between two networks, indicates the overall accuracy of this methodology, whileRMS_{S0} indicates the residuals in each point after transformation, using measurement of a certain day, calculated with a certain strategy. S_{C0} is the standard deviation of C_0 of all the sessions in one day. It is the precision of a strategy, i.e., an indicator of the repeatability. The smaller S_{C0}, the more repeatable the strategy is, and vice versa. The difference between the maximum and minimum, and the mean value of C_0 ($\overline{C_0}$) reveal the tendency of bias and systematic error in the result because $\overline{C_0}$ → 0 if the error is normally distributed. However, some systematic errors in this study are non-linear (e.g. errors in the antenna model and geoid model) and thus incapable of being eliminated with applied methodology, see Chapter 5.3.4 and 5.4. for details. Therefore, C_0 is not expected free from systematic error, un-biased and normally distributed. $\overline{C_0}$ is an indicator of the magnitude of such non-linear error. Sample size is relatively small in this study, it is impossible to eliminate all the affection of random errors in $\overline{C_0}$. Thus, $\overline{C_0}$ tells the direction of systematic error and the absolute height deviation mainly due to systematic errors, but not the value of it.

In this study, S_0 and C_0 is computed with Gtrans simultaneously with the transformation. They are saved as an individual file for each of the two transformations of each session. Other indicators are subsequently computed with Microsoft Excel.

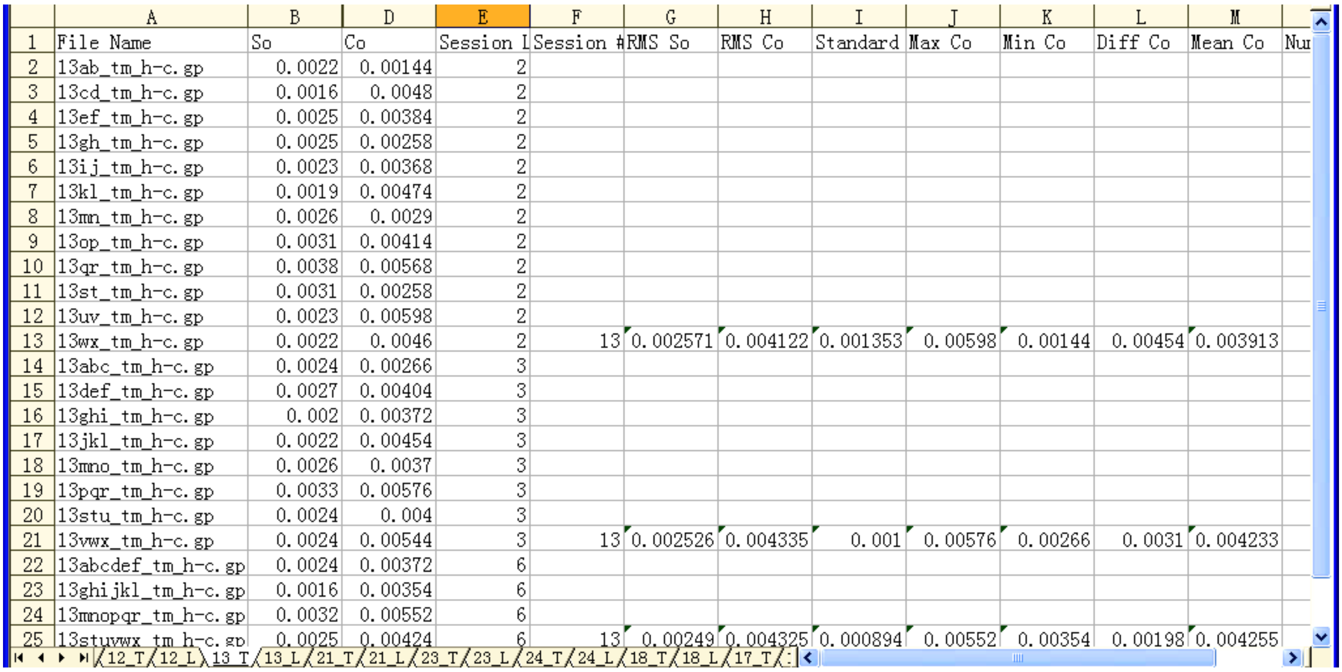
Practical Procedure of statistics

Due to the complexity of file arrangement and Excel operation, an Excel VBA macro is developed to automate some procedures of statistics (see Appendix 2). It is composed of five relatively independent subroutines (See Figure. 6), including:

1. check the input files of statistics: read input files, creating corresponding worksheets and fill in data

2. calculate indicators of all the worksheets
3. calculate a single worksheet
4. generate the table of result.

With the macro, the input files of statistics are firstly automatically checked to see if it is the desired one, i.e., if it was generated with the correct transformation. If any trace of error found, the relevant transformation must be re-computed in Gtrans. With correct input files of statistics, S_0 in the first transformation, as well as S_0 and C_0 in the second transformation are loaded. They are arranged with the step of transformation, day, session duration and strategy applied and subsequently filled into their corresponding worksheets. One worksheet is created for each step of transformation of a strategy, i.e., for each strategy, two worksheets, separate for "network constraint" and for "check with known network" are created. Then, statistical indicators are calculated in each worksheet and eventually organized into the table of result (Table 8). Figure 6 shows the general structure of such Excel worksheet for statistics. If any error was found in a single worksheet, the problem must be solved and that single worksheet can be calculated individually after modifications and the table of result can be generated again. See Figure 7.



	A	B	D	E	F	G	H	I	J	K	L	M	
1	File Name	So	Co	Session l	Session #	RMS So	RMS Co	Standard	Max Co	Min Co	Diff Co	Mean Co	Nu
2	13ab_tm_h-c.gp	0.0022	0.00144	2									
3	13cd_tm_h-c.gp	0.0016	0.0048	2									
4	13ef_tm_h-c.gp	0.0025	0.00384	2									
5	13gh_tm_h-c.gp	0.0025	0.00258	2									
6	13ij_tm_h-c.gp	0.0023	0.00368	2									
7	13kl_tm_h-c.gp	0.0019	0.00474	2									
8	13mn_tm_h-c.gp	0.0026	0.0029	2									
9	13op_tm_h-c.gp	0.0031	0.00414	2									
10	13qr_tm_h-c.gp	0.0038	0.00568	2									
11	13st_tm_h-c.gp	0.0031	0.00258	2									
12	13uv_tm_h-c.gp	0.0023	0.00598	2									
13	13wx_tm_h-c.gp	0.0022	0.0046	2	13	0.002571	0.004122	0.001353	0.00598	0.00144	0.00454	0.003913	
14	13abc_tm_h-c.gp	0.0024	0.00266	3									
15	13def_tm_h-c.gp	0.0027	0.00404	3									
16	13ghi_tm_h-c.gp	0.002	0.00372	3									
17	13jkl_tm_h-c.gp	0.0022	0.00454	3									
18	13mno_tm_h-c.gp	0.0026	0.0037	3									
19	13pqr_tm_h-c.gp	0.0033	0.00576	3									
20	13stu_tm_h-c.gp	0.0024	0.004	3									
21	13vwx_tm_h-c.gp	0.0024	0.00544	3	13	0.002526	0.004335	0.001	0.00576	0.00266	0.0031	0.004233	
22	13abcdef_tm_h-c.gp	0.0024	0.00372	6									
23	13ghijkl_tm_h-c.gp	0.0016	0.00354	6									
24	13mnopqr_tm_h-c.gp	0.0032	0.00552	6									
25	13stuvwx_tm_h-c.gp	0.0025	0.00424	6	13	0.00249	0.004325	0.000894	0.00552	0.00354	0.00198	0.004255	

Figure 6. *A screenshot of a worksheet of statistics: the transformation of "check with known network" using strategy "D1_L1_1000"*

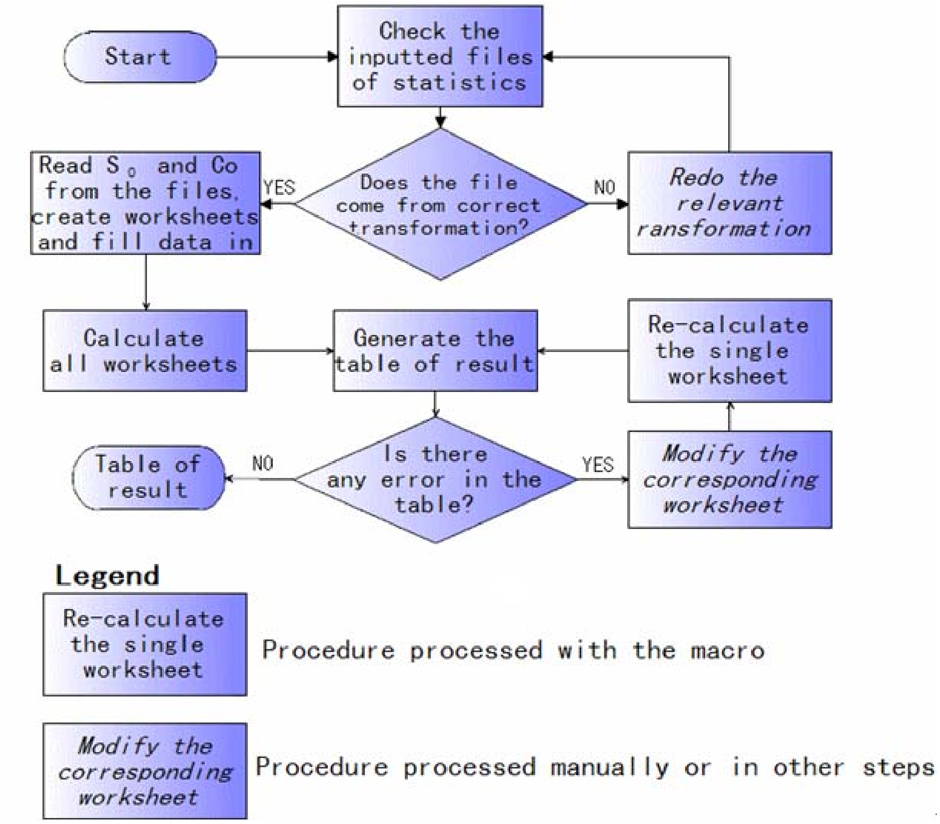


Figure 7. *Flowchart of the general progress of statistics*

The table of result is re-organised for better visualization into Table 8. Based on the resulting indicators, analysis is subsequently undertaken, which will be illustrated in Chapter 5.

5 Result and analysis

The GPS obtained normal heights in RH 2000 and the statistical indicators constitute the result of this study. The latter reveals the feasibility and accuracy of this methodology and is therefore of outmost interest. The results are summarized in Table 8.

In 2008, Lantmäteriet performed a comparable analysis based on the same GPS measurement, using different software and settings. GeoGenius from TerraSAT GmbH was applied for GPS measurement processing, using broadcast orbit. Many processing time interval were tried, including 5 seconds, 15 seconds and 30 seconds while processing interval of 15-second is uniformly applied in this study, see Table 3. The former geoid model SWEN05_RH2000 was used in the analysis from 2008. Moreover, two different methods of network fitting were tested in the former calculation, denoted as "fitting" and "fixed". The former is the same method applied in this study (see Chapter 3.1 and 3.3), i.e., free network adjustment with one point fixed and then constrained to the known network by 1-dimentional 3-parameter transformation. The latter is to assign the correct heights to all the included benchmarks of the national network during network adjustment, rather than performing a free network adjustment and then a transformation. Therefore, the GPS-determined network is directly constraint (fixed) to the national network with network adjustment. See Chapter 5.3.4. The results of this former analysis of the same observation campaign are listed in Table 9 and 10. They are discussed together in this study in order to analyze the factors effecting the accuracy with more samples.

Table 8. *The statistics of the calculation in this study*

Session Name	Day	Session Length (h)	Number of Sessions	RMS of network fitting ($RMSs_0$, mm)	Local network error (mm)						
					RMS of height error ($RMSc_0$)	Standard deviation (Sc_0)	Max(C_0)	Min(C_0)	Max - min	Mean	RMS of standard error of unit weight ($RMSs_0$)
Leica antenna, L1 11 points, 6 receivers (11s6r)	1+2	3	15	7.8	5.5	4.1	11.2	-3.1	14.3	3.8	2.8
Leica antenna, L1 9 points, 6 receivers (9s6r)	1+2	3	15	5.7	7.0	6.5	12.6	-10.5	23.1	3.0	2.8
Leica antenna, LC 9 points, 6 receivers (9s6r_c)	1+2	3	16	2.6	3.8	2.7	7.0	-2.9	9.9	2.8	4.3
Leica antenna, L1, 20 points (D1_D2)	1	2	12	3.9	2.9	1.1	4.2	1.0	3.2	2.7	2.5
		3	8	3.8	2.9	0.8	3.8	1.5	2.3	2.8	2.4
		6	4	4.3	2.8	1.6	4.5	0.6	3.9	2.5	3.3
	2	2	12	3.5	4.3	1.0	5.9	2.7	3.2	4.2	2.3
		3	8	3.7	4.6	1.3	5.9	2.2	3.8	4.5	2.4
		6	4	6.7	5.1	2.6	6.4	0.7	5.7	4.6	3.9
Leica antenna, L1, 11 points (D1_L1_1000, D2_L1_1000)	1	2	12	3.4	4.1	1.4	6.0	1.4	4.5	3.9	2.6
		3	8	2.8	4.3	1.0	5.8	2.7	3.1	4.2	2.5
		6	4	2.6	4.3	0.9	5.5	3.5	2.0	4.3	2.5
	2	2	12	2.5	5.6	0.9	6.7	4.0	2.8	5.5	2.4
		3	8	1.9	5.5	0.8	6.1	4.1	2.1	5.5	2.4
		6	4	1.9	5.4	0.7	5.9	4.4	1.5	5.3	2.3
Leica antenna, Lc, 11 points (D1_Lc_1000, D2_Lc_1000)	1	3	7	2.6	4.1	1.4	5.5	1.9	3.6	3.9	3.1
	2	3	8	2.2	5.2	0.9	6.7	3.9	2.8	5.1	3.9
Leica antenna, L1, 11 points, points of 2000-series are used as known (D1_D2_L1_2000)	1	3	8	5.8	2.7	1.3	4.1	0.0	4.1	2.4	2.4
	2	3	7	5.8	4.7	1.5	7.0	2.4	4.5	4.5	2.3

DM antenna, L1, 11 points (D3_D4)	3	2	11	5.0	8.8	1.6	12.7	7.0	5.8	8.7	4.6
		3	8	4.7	8.6	1.0	10.1	7.4	2.7	8.5	4.5
		6	4	4.3	6.9	1.5	8.2	4.9	3.4	6.8	4.7
	4	2	12	3.4	9.0	1.9	10.5	3.6	6.9	8.8	4.3
		3	8	3.2	9.0	1.8	11.1	5.2	5.9	8.8	4.2
		6	4	3.9	6.4	2.4	9.3	3.8	5.5	6.0	4.6
DM antenna, Lc, 11 points (D3_Lc, D4_Lc)	3	3	8	4.3	7.8	0.9	9.3	6.2	3.1	7.8	4.4
	4	3	8	3.6	8.2	1.2	10.4	7.1	3.2	8.1	4.5

Note: See Chapter 4.1 for the settings of a strategy. See Chapter 4.3.1 and 5.1 for explanation of the statistical measures in this table.

Table 9. *The statistics of the former calculation in spring 2008, Day 1 and Day 2 with Leica antenna*

Method	Day	Sess. length (h)	Processing interval (s)	Number of sessions	RMS of network fitting (mm)	Local Network Error (mm)					
						RMS of height error ($RMSc_0$)	Standard divation (Sc_0)	Max(C_0)	Min(C_0)	max – min	RMS of standard error unit weight ($RMSs_0$)
Fixed	1	1	5	23		4.6	1.5	-1.4	-7.8	6.4	3
Fixed	1	2	5	12		4.2	1.1	-1.9	-5.7	3.8	1.8
Fixed	1	3	5	8		4.5	1.3	-2.9	-6.2	3.3	1.6
Fixed	1	3	15	8		4.1	1.1	-2.9	-6.2	3.3	1.6
Fixed	1	6	5	4		4.5	0.6	-3.8	-5.3	1.5	1.9
Fixed	1	6	15	4		3.9	0.5	-3.4	-4.5	1.1	1.6
Fixed	1	6	30	4		3.9	0.5	-3.3	-4.4	1.2	1.5
Fitting	1	1	5	24	5.1	5	1.4	-2.4	-7.6	5.2	2.4
Fitting	1	2	5	12	4.8	5	1.1	-3.2	-6.2	3	2.4
Fitting	1	3	5	8	4.7	5.3	1	-4	-6.4	2.4	2.1
Fitting	1	3	15	7	4.7	4.9	1	-3.8	-6.2	2.4	2.2
Fitting	1	6	5	4	4.8	5.4	0.4	-5	-6	1	2.3
Fitting	1	6	15	4	4.7	5	0.3	-4.6	-5.2	0.6	2.1
Fitting	1	6	30	4	4.7	4.9	0.3	-4.4	-5.2	0.8	2.1
Fitting	2	1	5	24	4.4	6.3	1.1	-4.6	-8.4	3.8	2.5
Fitting	2	2	5	12	4.8	6.1	0.8	-5	-7.4	2.4	2.3
Fitting	2	3	5	8	4.0	6.1	0.3	-5.6	-6.4	0.8	2
Fitting	2	3	15	8	4.0	6	0.3	-5.6	-6.4	0.8	2
Fitting	2	6	5	4	4.0	6.4	0.5	-5.8	-6.8	1	2.1
Fitting	2	6	15	4	4.0	6.6	0.2	-6.4	-6.8	0.4	2.1
Fitting	2	6	30	4	4.0	6.6	0.3	-6.2	-6.8	0.6	2.1

Note: See Chapter 5.3.4 for details about "fixed" and "fitting" method. See Chapter 4.3.1 and 5.1 for explanation of the statistical measures in this table.

Table 10. *The statistics of the calculation in spring 2008, Day 3 and Day 4 with Dorne Margolinand Javad antenna*

Method	Day	Sess. length (h)	Processing interval (s)	Number of sessions	RMS of network fitting (mm)	Local Network Error (mm)					
						RMS of height error (RMSc0)	Standard divation	Max(C_0)	Min(C_0)	Diff max – min	RMS of standard error unit weight ($RMSs_0$)
Fixed	3	1	5	24		8.2	1.7	-5.7	-11.4	5.7	2.4
Fixed	3	2	5	12		7.9	1.4	-5.7	-11	5.3	2.3
Fixed	3	3	5	8		7.6	1	-5.9	-8.7	2.8	2.2
Fixed	3	3	15	8		7.7	1	-6	-8.7	2.7	2.2
Fixed	3	6	5	4		7.9	0.7	-6.8	-8.4	1.6	2.2
Fixed	3	6	15	4		7.9	0.7	-6.8	-8.4	1.6	2.2
Fixed	3	6	30	4		7.9	0.7	-6.9	-8.4	1.5	2.3
Fixed	4	1	5	24		8.9	2.8	-4.9	-14.6	9.7	2.5
Fixed	4	2	5	12		8.3	1.9	-5.5	-11.3	5.8	2.3
Fixed	4	3	5	8		8.3	1.5	-5.9	-11.2	5.3	2.2
Fixed	4	3	15	8		8.3	1.5	-5.9	-11.2	5.3	2.3
Fixed	4	6	5	4		8.2	0.6	-7.6	-9	1.4	2.3
Fixed	4	6	15	4		8.2	0.6	-7.6	-9	1.4	2.3
Fixed	4	6	30	4		8.1	0.6	-7.5	-9	1.5	2.3
Fitting	3	1	5	24	3.3	9.9	1.7	-7.6	-14.2	6.6	3.2
Fitting	3	2	5	12	2.7	9.7	1.3	-8.4	-13.2	4.8	3.3
Fitting	3	3	5	8	2.5	9.5	0.8	-8.6	-11.2	2.6	3
Fitting	3	3	15	7	2.6	9.6	0.8	-8.6	-11.2	2.6	3
Fitting	3	6	5	4	2.5	9.7	0.6	-9	-10.2	1.2	3.2
Fitting	3	6	15	4	2.4	9.7	0.5	-9	-10.2	1.2	3.1
Fitting	3	6	30	3	2.5	9.8	0.7	-8.8	-10.4	1.6	3.2
Fitting	4	1	5	24	3.4	10.9	2.7	-6.4	-16.8	10.4	3.4
Fitting	4	2	5	12	2.8	10.2	1.7	-6.4	-12.4	6	3.2
Fitting	4	3	5	8	2.4	10.2	1.4	-8	-12.4	4.4	3.1
Fitting	4	3	15	8	2.4	10.2	1.4	-8	-12.4	4.4	3.2
Fitting	4	6	5	4	2.3	10.1	0.8	-9	-10.6	1.6	3.2
Fitting	4	6	15	4	2.3	10	0.7	-9	-10.6	1.6	3.3
Fitting	4	6	30	4	2.3	10.1	0.8	-9	-10.8	1.8	3.2

Note: See Chapter 5.3.4 for details about "fixed" and "fitting" method. See Chapter 4.3.1 and 5.1 for explanation of the statistical measures in this table.

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5.1 Some explanation on the tables of result

The table of statistics of this study, Table 8, can be understood as three components. Firstly, the first column to the fourth column illustrates the data used and strategy applied in that trial of computation. Secondly, the fifth column, RMS of network fitting , lists RMS of standard error of unit weight (RMS_{S_0}) in the first transformation, i.e., free network constraint. Thirdly, the sixth column to the last column is indicators from the second transformation, i.e., comparing with the pre-determined network.

The tables of statistics of the former analysis, Table 9 and 10, have very similar structure. As mentioned before, there were two methods of network fitting tried in that study, they are distinguished with "fixed" and "fitting" in the first column.

5.2 The overall accuracy

In Table 8, 9 and 10, the RMS of C_0 (RMS_{C0}) varies from 2.7 to 10.9 mm with different strategy applied. The mean RMS_{C0} is 6.7 mm and in almost all the 76 cases, it achieves 1-cm accuracy except 6 of them. In 24 trials, the 5-mm accuracy is achieved. Therefore, conservatively, this method reaches 1-cm accuracy. See Figure 8.The conservative estimate of accuracy is proposed here in spite that the mean of RMS of C_0 is 6.7 mm. That is because firstly, the conditions for observations in the study area are good, as mentioned before in Chapter 2. Thus the GPS measurement is more likely to reach higher accuracy. Secondly, the geoid model here is fairly flat and have good accuracy. In other parts of Sweden, the mean accuracy of this proposed method thus might be worse than 6.7 mm. Thirdly, the height error in a certain session may vary around RMS_{C0}.

The standard deviation of the height error, S_{C0}, indicates the precision of this methodology. In Table 8, 9 and 10, it varies from 0.3 mm to 2.7 mm (1-σ level) with different trials in most (74) cases. In two extreme cases, S_{C0} reach peaks of 4.1 and 6.5 mm. That means even the mean of RMS_{co}, 6.7 mm, (which is affected by the extreme values) was concerned as the expected C_0, and 2.7 mm was assumed as its standard deviation, there is still two third of the results having better accuracy than 9.4 mm.

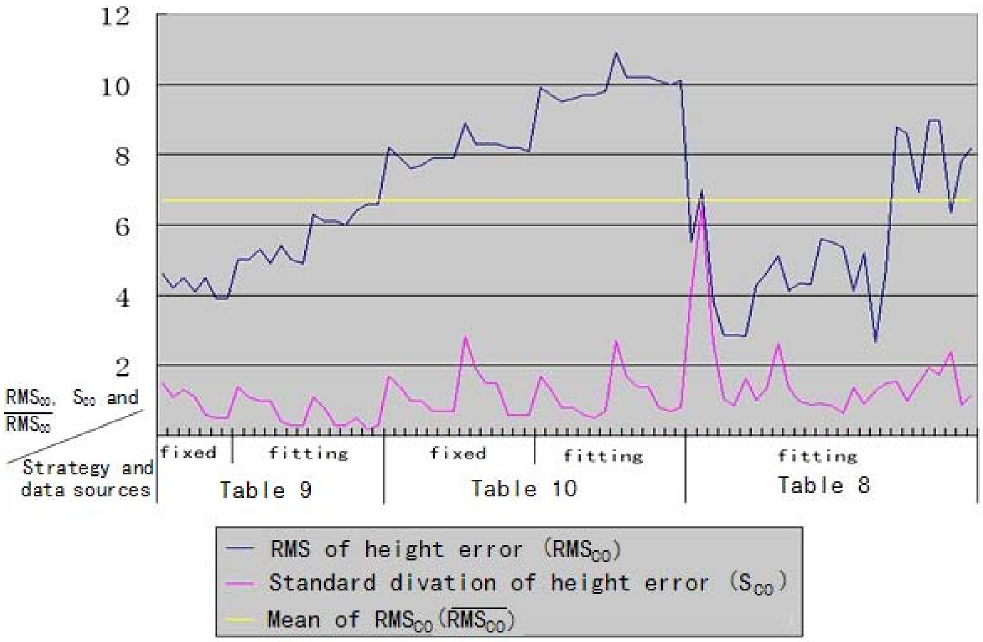


Figure 8. *RMS and standard deviation of height errors*

To achieve this level of accuracy, the RMS of network fitting in the first transformation (RMS_{so}) varies from 1.9 mm to 7.8 mm, i.e., the final accuracy might not been achieved if the GPS measurement is worse. Moreover, the RMS_{so} in the second transformation is from 1.6 mm to 4.7 mm besides C_0.

In conclusion, this result reveals that with static GPS observation, with commercial GPS software and SWEN 08 as geoid model, it is feasible to achieve 1 cm accuracy (1σ) in height determination in major part of Sweden.

5.3 Accuracy with regards to different factors

This chapter focus on the impact of a single factor upon the quality, especially the accuracy and repeatability, of this method. Different options of a parameter are compared while other parameters are kept fixed, to ensure that the result is affected only by the analyzed factor.

Session duration

The impact of session duration is evaluated by comparing statistical indicators computed using 1-hour, 2-hour 3-hour and 6-hour observation. Only comparable strategies are compared together to ensure that all the other parameters except session duration are identical. The comparison is performed by using an indicator of shorter observation time minus the same indicator of longer observation time: the RMS_{C0} or S_{C0} of 1-hour observation minus the same indicator of 2-hour observation, etc. The differences shows the tendency of change. In Table 11, the comparison using statistics of the former study is given. Table 12 is the same comparison using statistics of this study. For a better visualization, Figure 9 and Figure 10 is plotted out of Table 11 and Table 12 separately.

Table 11. *The impact of observation time on accuracy (study in 2008), Unit: mm*

Method	ΔRMS_{Co} 1h-2h	ΔRMS_{Co} 2h-3h	ΔRMS_{Co} 3h-6h	ΔS_{Co} 1h-2h	ΔS_{Co} 2h-3h	ΔS_{Co} 3h-6h
Fixed Day 1	0.4	-0.3	0	0.4	-0.2	0.7
Fitting day 1	0	-0.3	-0.1	0.3	0.1	0.6
Fitting day 2	0.2	0	-0.3	0.3	0.5	-0.2
fixed Day 3	0.3	0.3	-0.3	0.3	0.4	0.3
Fixed Day 4	0.6	0	0.1	0.9	0.4	0.9
fitting Day 3	0.2	0.2	-0.2	0.4	0.5	0.2
fitting Day 4	0.7	0	0.1	1	0.3	0.6
Mean	0.3	-0.01	-0.1	0.5	0.3	0.4

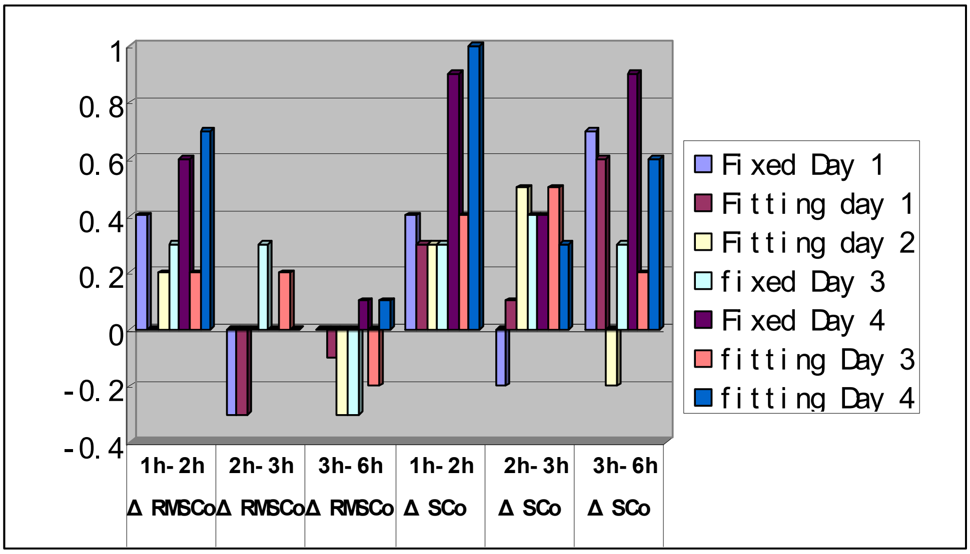


Figure 9. *The effect of observation time to accuracy (calculation of 2008), Unit mm*

Table 12. *The effect of observation time to accuracy (calculation of this study), Unit mm*

Session Name	ΔRMS_{Co} 2h-3h	ΔRMS_{Co} 3h-6h	ΔS_{Co} 2h-3h	ΔS_{Co} 3h-6h
Day 1, L1	0.0	0.0	0.2	-0.8
Day 2,L1	-0.3	-0.5	-0.3	-1.3
Day 1, L1, 11 known sites	-0.2	0.0	0.4	0.1
Day 2,L1, 11 known sites	0.1	0.1	0.1	0.2
Day 3, L1	0.2	1.6	0.6	-0.5
Day 4, L1	0.0	2.6	0.2	-0.6
Mean	0.0	0.7	0.2	-0.5

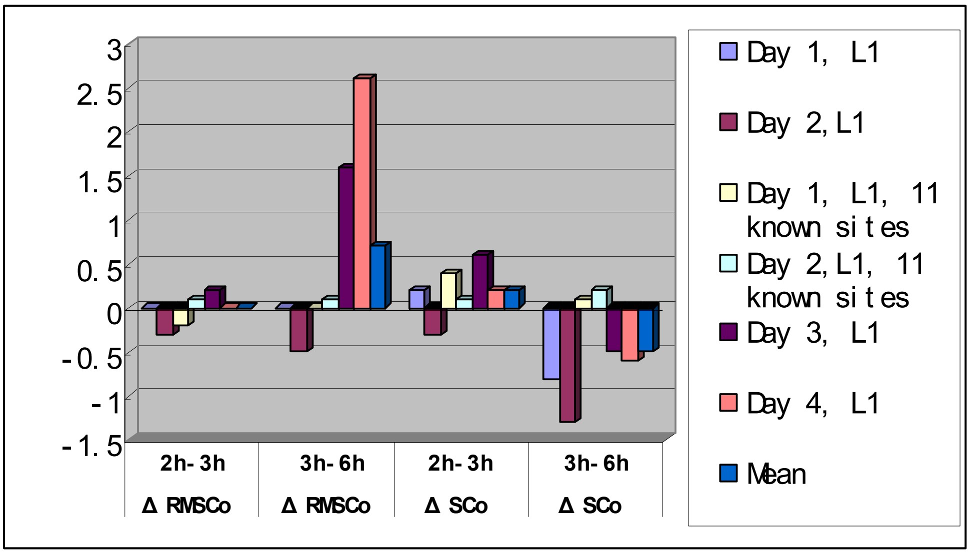


Figure 10. *The affection of observation time to accuracy (calculation of this study), Unit mm*

The following tendency can be observed in Table 11 and Table 12: Firstly, with the increase of session duration from 1 hour to 2 hours, the overall accuracy, precision and repeatability is improved in all the four days. Secondly, with the further increasing of observation time to 3 hours, in the former study (Table 11), RMS$_{Co}$ increased rather slightly by 0.01 mm, however, S$_{Co}$ decreased at a comparatively larger scale by 0.3 mm, showing a better repeatability. Therefore, comprehensively, 3-hour observation yields a better result practically in this case. In the statistics of this study (Table 12), the accuracy is the same but the repeatability is better with 3-hour observation. Thirdly, with the session duration was further increased to 6 hours, the similar phenomenon exists as it was increased to 3 hours in Table 11. However, in Table 12, the mean RMS$_{Co}$ decreased by 0.7 mm while the mean S$_{Co}$ increased by 0.5 mm. So practically in this case, there is insignificant improvement. Moreover, in the statistics of the former study (Table 9 and 10) the absolute value of maximum or minimum C$_0$, and the differences between them are usually smaller in the results of longer session length, showing that the precision of the longer observation is usually better. This tendency also generally exists in the statistics of this study (Table 8) in spite of more exceptions.

As to the absolute error (C$_0$ in the second transformation), according to Table 9 and 10, some results of 1-hour measurements are so inaccurate and unstable that they fail to fit the 1-cm accuracy level. For example, almost all the results in Day 3 and Day 4 of 1-hour observation are thought unacceptable. Therefore, it was concluded that 1-hour observation time is too short for normal height determination of height system. This is also the reason why the 1-hour observation was not applied in this study. As to the 2-hour observation, in the former analysis, the results of Day 4, and the results of Day 3 using fitting method are still thought too inaccurate (see Table 10), but most results of 2-hour observation are sufficient. The similar tendency exists in this study: the results of 2-hour measurement in Day 3 and Day 4 using L1 (two trials) are found fail to achieve 1-cm accuracy, however, most of results using 2-hour measurement are acceptable except these two individual cases.

In conclusion, it is found in the results of both studies that longer observation time (session duration) tends to yields better results. However, it is just a general tendency. The increase of accuracy and repeatability is not linear and not proportional with the increase of session duration. Practically, there are even some exceptions. 2 hours is found acceptable. Therefore, considering the expense and inefficiency of long observation time, 2 hours or 3 hours are proposed for application.

Wavelength

There are two wavelengths applied in this study: L1 and the ionospheric-free linear combination, denoted as Lc. As the effect of ionosphere is frequency depended, it is thus possible to "eliminate the ionosphereric refraction by using two signals with different frequency" (Hofmann-Wellenhof et al., 1997). Lc is a commonly used linear combination of L1 and L2 for such purpose. In carrier phase observables,

$$Lc= (f_1^2L1-f_2^2L2)/(f_1^2-f_2^2) \qquad (12)$$

where f_1 and f_2 are the frequency of L1 and L2 (Hofmann-Wellenhof et al., 1997). However, the so called "ionosphere-free" cannot remove all the ionospheric disturbances because there are some approximation involved, see Hofmann-Wellenhof et al. (1997). Meanwhile, Lc amplifies any noise in L1 or L2 by 3 times. In addition, it is argued that in local scale where the baselines are less than few tens of kilometres, the ionospheric delays are identical for all receives and thus can be ignored (eg. Colombo, 2000). Therefore, L1 is proposed by Lantmäteriet rather than Lc in GPS surveying of smaller network mentioned in "HMK Geodesi GPS" (Lantmäteriet, 1996). However, in this study, the baseline is longer than the recommendation (Lantmäteriet, 1996) and the consideration of ionospheric affection is thus thought necessary. Also, note that this recommendation is based on the experience some 15 years ago, and may therefore be re-considered (see Chapter 2.1).

With the larger network in this study (see Chapter 2.1), the comparison between L1 and Lc (See Table 13) shows that Lc yields similar or even better accuracy. This is indicated in Table 13 that the RMS of network error (RMS_{Co}) using L1 is always larger than using Lc. The strategy 9s6r using L1 was found having poor accuracy (See Chapter 5.2 and 5.3.1), but the equivalent strategy 9s6rc using the same method but using Lc signal is found much better. Except this extreme example, RMS_{Co} using Lc is still smaller by 0.2 to 0.8 mm. But their repeatability, Sco, is similar. Moreover, the mean of C_0 ($\overline{C_0}$) using Lc are found always smaller, showing the measurements using Lc contain less systematic error than using L1. RMS_{So} in the first transformation (fitting to known network) using Lc is similar, and slightly worse than L1 except the extreme example of 9s6rc. RMS_{So} of the second transformation (compare the points of 9000-series to their known height) using Lc is much worse than using L1 (See Table 13). That means the systematic error of the network using Lc is smaller, but the error in each individual site is larger. This phenomenon coincides with the theory that Lc partly eliminates ionosphereric affection but amplify the noise.

Table 13. *A comparison of the results between L1 and Lc, session duration of 3 hours*

Session	Diff. RMS of Network fitting* (ΔRMSso, mm)	Diff. local height error* (mm)				
		ΔRMSco	ΔSco	Δ(Max-Min)	ΔMean	ΔRMSso
Leica antenna, 9 sitess, 6 receivers	3.1	3.2	3.8	13.3	0.3	-1.4
Leica antenna, D1, 11 points	0.1	0.2	-0.4	-0.5	0.3	-0.6
Leica antenna, D2, 11 points	-0.3	0.3	-0.1	-0.7	0.3	-1.5
DM antenna, D3, 11 points	0.4	0.8	0.1	-0.4	0.8	0.2
DM antenna, D4, 11 points	-0.5	0.8	0.6	2.6	0.7	-0.3
Mean value except 9s6r & 9s6rc	-0.08	0.53	0.05	0.25	0.53	-0.55

Note: the differences are calculated by the indicators using L1 minus the corresponding indicators using Lc.

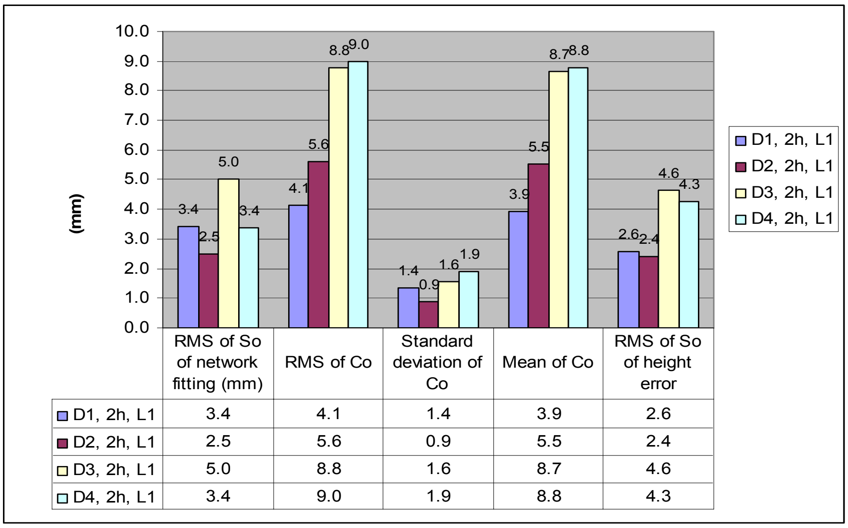
In conclusion, this study shows that it has the potential to achieve similar or practically even better accuracy with Lc combination, which is contrary with the former proposal. However, it is not enough to conclude that Lc is better than L1 here with only four pairs of comparisons and measurement of 4 days at the same time of a year. The following elements might effect the choice of wavelength: firstly the length of baseline: it is argued that with short baselines, the atmospheric effect is insignificant (Colombo, 2000). Secondly, the magnitude and condition of ionosphereric refraction, which directly determined by the intensity of ionization, see Chapter 6.2.2.

Thus, based on relevant study and the default settings of the software TTC, it is suggested that L1 is better for baselines less than 5 km; while Lc is proposed for baselines longer than 5 km. As to the effect of time, more study on the ionosphere is needed.

Antenna

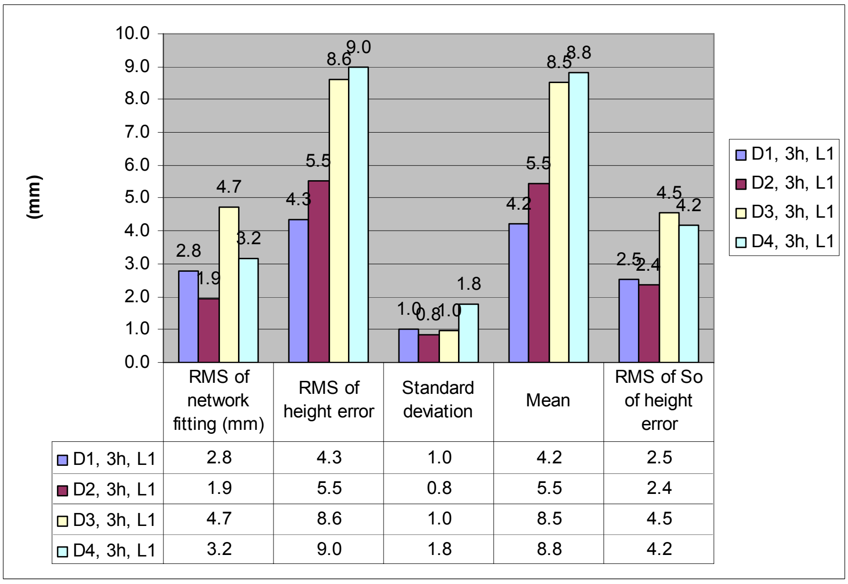
There are three types of antennas applied in this study: Leica LEIAX1202GG is used on the first and the second day on all sites. On the third and the forth day, Javad JNSCR_C-146-22-1 and Ashtech copies of the Dorne Margolin Model T are both applied on different sites as listed in Table 2. These choke-ring antennas with Dorne Margolin antenna elements are expected to get better result as they are more scientific antennas. However, the result turns to be unexpected that commercial standard antennas (here Leica LEIAX1202GG) gives better result.

In the analysis in 2008, by comparing Table 9, the result of Day 1 and 2 with Leica antenna, with Table 10, the result of Day 3 and 4 using choke-ring antennas, it is obvious that the former is much better. In Day 1 and Day 2, the RMS of network errors (RMS_{C0}) varies from 3.9 to 4.6 mm with the mean value 5.2 mm. However, in Day 3 and 4, it varies from 7.6 to 10.9mm with the mean value 8.8 mm. The accuracy of measurements using standard commercial antenna is much better in this case. Standard deviation of height error of Day 1 and Day 2 is slightly better in most of strategies applied, showing slightly better repeatability using Leica antenna. Moreover, the RMS of network fitting in the first transformation (RMS_{S0}) is also much smaller using Leica antenna, showing the GPS measurement using Leica antenna is better, as well as the RMS_{S0} in the second transformation in most strategies. In the computation of this study, i.e. Table 8, because the statistics is arranged by strategies rather than by date, they are therefore re-organized from Table 8 and plotted in Figure 11, 12, 13, and 14, comparing results of different days while the other parameters remain. The results of the first two days' measurement with Leica antenna is still found better, and the general tendency of each indicator is similar as it is in the former calculation.



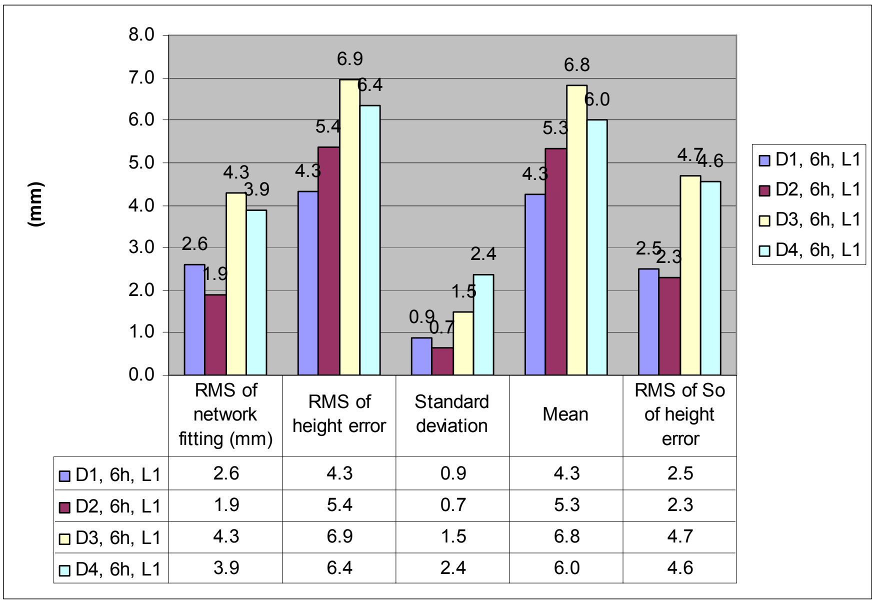
	RMS of So of network fitting (mm)	RMS of Co	Standard deviation of Co	Mean of Co	RMS of So of height error
D1, 2h, L1	3.4	4.1	1.4	3.9	2.6
D2, 2h, L1	2.5	5.6	0.9	5.5	2.4
D3, 2h, L1	5.0	8.8	1.6	8.7	4.6
D4, 2h, L1	3.4	9.0	1.9	8.8	4.3

Figure 11. *A comparison between the four days using 2-hour session length and signal of L1, using the computation of this study (Standard antennas are used day 1 and 2, choke ring antennas are used day 3 and 4.)*



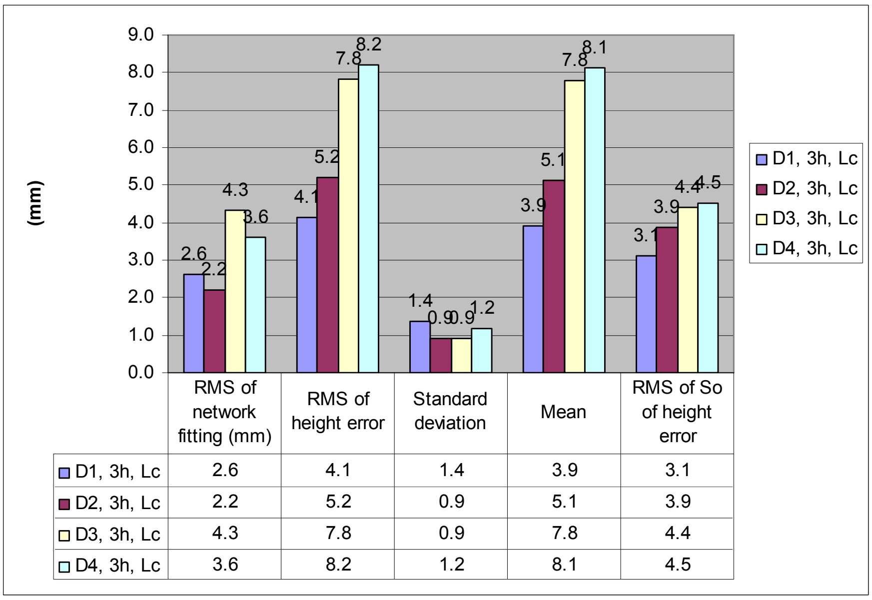
	RMS of network fitting (mm)	RMS of height error	Standard deviation	Mean	RMS of So of height error
D1, 3h, L1	2.8	4.3	1.0	4.2	2.5
D2, 3h, L1	1.9	5.5	0.8	5.5	2.4
D3, 3h, L1	4.7	8.6	1.0	8.5	4.5
D4, 3h, L1	3.2	9.0	1.8	8.8	4.2

Figure 12. *A comparison between the four days using 3-hour session length and signal of L1, using the computation of this study (Standard antennas are used day 1 and 2, choke ring antennas are used day 3 and 4.)*



	RMS of network fitting (mm)	RMS of height error	Standard deviation	Mean	RMS of So of height error
D1, 6h, L1	2.6	4.3	0.9	4.3	2.5
D2, 6h, L1	1.9	5.4	0.7	5.3	2.3
D3, 6h, L1	4.3	6.9	1.5	6.8	4.7
D4, 6h, L1	3.9	6.4	2.4	6.0	4.6

Figure 13. *A comparison between the four days using 6-hour session length and signal of L1, using the computation of this study (Standard antennas are used day 1 and 2, while choke ring antennas are used day 3 and 4.)*

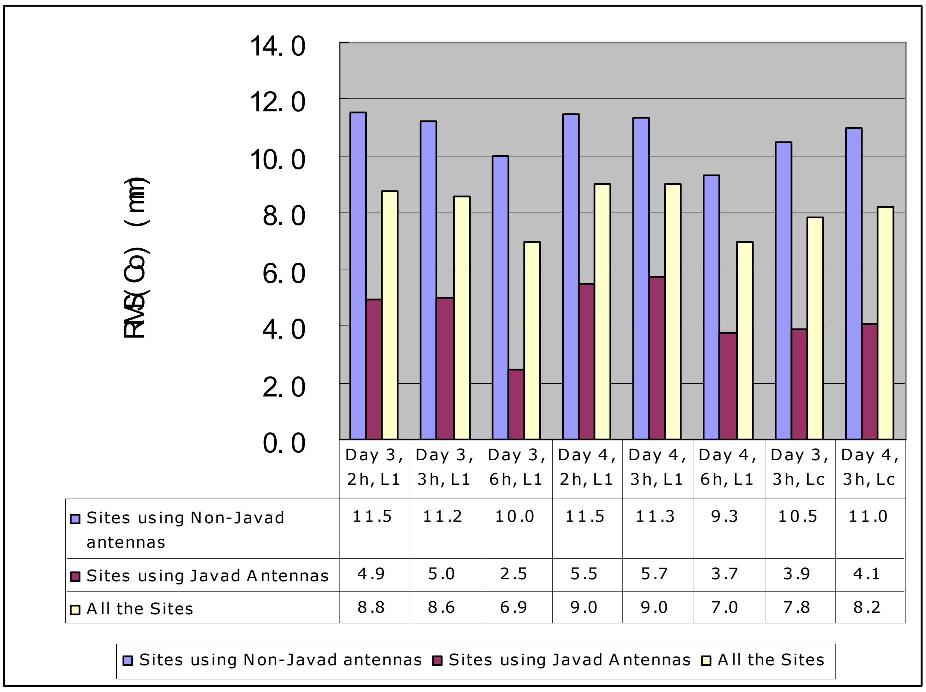


	RMS of network fitting (mm)	RMS of height error	Standard deviation	Mean	RMS of So of height error
D1, 3h, Lc	2.6	4.1	1.4	3.9	3.1
D2, 3h, Lc	2.2	5.2	0.9	5.1	3.9
D3, 3h, Lc	4.3	7.8	0.9	7.8	4.4
D4, 3h, Lc	3.6	8.2	1.2	8.1	4.5

Figure 14. *A comparison between the four days using 3-hour session length and signal of Lc, using the computation of this study (Standard antennas are used day 1 and 2, while choke ring antennas are used day 3 and 4.)*

Potential errors existing in the antenna model is found the primary reason for this “unexpected” result. According to the argument by Eriksson (2009), because both types of antenna used in the last two days’ measurement are different "copies" of the classic AOAD/M_T. They are both modelled from the known relative calibration models. However, due to different manufacturers, there might be a residual effect of the "AOAD/M_T-copies" among them (Eriksson, 2009).

An additional comparison is performed between points measured with Javad and "non-Javad" antennas in Day 3 and Day 4 (See Figure 15). The C_0 of Points 9001, 9002 and 9004, on which non-Javad antennas were used, and Points 9003 and 9005, on which Javad antennas were used, are separately compared to their pre-determined normal heights following Equation (7). The RMS_{C0} of the two groups are calculated following Equation (8), as the same procedure to calculate other RMS_{C0} in Table 8. Figure 15 is the result and the plot of this test. It is obvious that the accuracy using Javad antenna is much better than using non-Javad ones: non-Javad antennas cause 2 to 4 times errors of Javad antennas. It can be conclude that mistakes exist in the antenna model of the non-Javad antennas. The results of Day 3 and Day 4 are seriously disturbed because 3 out of 5 sites in the local network use non-Javad antenna in the last two days. It should also be noted that 5 out of 6 antennas in at known points are Javad antennas (Table 2). The RMS_{C0} use only Javad antennas can be found very close to their comparable strategies in Day 1 and Day 2 using Leica LEIAX1202GG. Thus, it can be conclude that the errors in antenna models cause the major distortion in the difference between Leica and non-Leica antennas. However, other reason cannot be excluded.



	Day 3, 2h, L1	Day 3, 3h, L1	Day 3, 6h, L1	Day 4, 2h, L1	Day 4, 3h, L1	Day 4, 6h, L1	Day 3, 3h, Lc	Day 4, 3h, Lc
Sites using Non-Javad antennas	11.5	11.2	10.0	11.5	11.3	9.3	10.5	11.0
Sites using Javad Antennas	4.9	5.0	2.5	5.5	5.7	3.7	3.9	4.1
All the Sites	8.8	8.6	6.9	9.0	9.0	7.0	7.8	8.2

Figure 15. *A comparison between the height error of points of 9000-series using Javad and Non-Javed antennas, using the computation of this study*

The differences on environment might also affect the result in Day 3 and Day 4. As mentioned in Chapter 2, the GPS measurements for four days were not performed at the same time of a year: the first two days are 18th to 20th in February, the third day is from 17th to 18th in March and the fourth day is 19th to 20th, March. Thus, many environmental elements differ. For example, during the first two days, the weather is stable but it snowed in the evening between 18th and 19th of March (See Eriksson, 2009 for more discussion on troposphere). Various of time dependent parameters of environment might affect GPS measurement according to relevant study, for example, the ionosphere dramatically varies with the sun, and "also dependent on the solar cycle, on events such as travelling ionospheric disturbances (TID) and, in general, on geomagnetic and 'space weather' conditions" (Colombo, 2000). Therefore, it is reasonable that considering the difference in environmental elements, the weather for example, might be a part of the reason of the "unexpected" results. Larger sample size is needed to study the existence and magnitude due to the differences of environment. Unfortunately, they cannot be discussed in detail in this study due to the subject of this study.

In conclusion, the measurements in Day 3 and Day 4 were mainly deformed by the unsuitable antenna models applied to the non-Javad antennas. Nevertheless, the affection of other possible elements, for example the troposphere condition, cannot be excluded. This study proved that it is possible to get fairly ideal result of normal height with GPS using modern commercial standard antennas, like the Leica LEIAX1202GG, and dual frequency GPS receivers. However, due to different manufacturers, possible deviations on the phase centre might occur on different variations of the same type of antenna. Therefore, the antennas should be well calibrated and the exact antenna phase centre variation (PCV) model from the correct manufacturer must be used if different antennas were used together in one session. Pratically, it might be difficult to find an exact PCV antenna model sometimes. Since the phase centre cannot be measured exactly and it is different for L1 and L2, it is therefore recommended to use the same type of antennas in one session. In such case, the systematic error can be considerably reduced even if an inaccurate antenna PCV model is applied – at least in networks of limited size.

Method of network fitting

In the former analysis in 2008, two methods of network fitting were tested, denoted separately as "fitting" and "fixed" method. The former treats the GNSS-obtained heights as a free network: free network adjustment is performed with only one point fixed after computing baselines. The free network would not been constrained to the known network of RH 2000 until the GNSS-determined ellipsoidal heights were converted into approximate normal heights (See Chapter 3). This method was based on the theory that GNSS observation has relatively high internal accuracy but might be tilt as a whole due to errors in GNSS measurement and in geoid model. Thus, it is better to firstly ensure the internal accuracy by free network adjustment, and then fit the internally good network to the known network to absorb the tilt. This is the only method of network fitting applied in this study. The latter, so called "fixed" method, constraints the GNSS-obtained network to the known network of RH 2000 directly in network adjustment using least square adjustment. Practically, it is realized by assigning the known heights to the benchmarks of national network included in the GPS measurement during network adjustment so that all the points in the national network are fixed in the network adjustment of the computed GPS baselines.

Theoretically, the "fitting" method is proposed for keeping the good internal accuracy and absorbing the tilt of the GNSS determined network (Hofmann-Wellenhof et al., 1997). Therefore, it is believed better and proposed (Lantmäteriet, 1996). However, according to the statistics of the former analysis in 2008, the "fixed" method yields even better accuracy. In Table 14, the indicators of using both methods are compared: two equivalent strategies with the same settings except the method of network fitting are compared. Indicators using "fitting" minus the indicators using "fixed", obtain their differences. Generally, the result shows that "fixed" method gives better overall accuracy (RMS_{C0}) for all the strategies, 1.5 mm by average. However, the differences of S_{C0} and (Max-Min) shows that free network fitting have similar, or even slightly better precision. Generally, this study shows the "fitting" method might not always be the optimum option.

***Table 14.** The comparison between two methods of network fitting (mm)*

Strategy(Day, session length, processing interval)	Δ RMS(C_0)	Δ S(C_0)	Δ (Max-min)	Δ RMS(S_0)
Day 1, 1 h, 5 s	0.4	-0.1	-1.2	-0.6
Day 1, 2 h, 5 s	0.8	0	-0.8	0.6
Day 1, 3 h, 5 s	0.8	-0.3	-0.9	0.5
Day 1, 3 h, 15 s	0.8	-0.1	-0.9	0.6
Day 1, 6 h, 5 s	0.9	-0.2	-0.5	0.4
Day 1, 6 h, 15 s	1.1	-0.2	-0.5	0.5
Day 1, 6 h, 30 s	1	-0.2	-0.4	0.6
(Mean of Day 1)	0.8	-0.2	-0.7	0.4
Day 3, 1 h, 5 s	1.7	0	0.9	0.8
Day 3, 2 h, 5 s	1.8	-0.1	-0.5	1
Day 3, 3 h, 5 s	1.9	-0.2	-0.2	0.8
Day 3, 3 h, 15 s	1.9	-0.2	-0.1	0.8
Day 3, 6 h, 5 s	1.8	-0.1	-0.4	1
Day 3, 6 h, 15 s	1.8	-0.2	-0.4	0.9
Day 3, 6 h, 30 s	1.9	0	0.1	0.9
(Mean of Day 3)	1.8	-0.1	-0.1	0.9
Day 4, 1 h, 5 s	2	-0.1	0.7	0.9
Day 4, 2 h, 5 s	1.9	-0.2	0.2	0.9
Day 4, 3 h, 5 s	1.9	-0.1	-0.9	0.9
Day 4, 3 h, 15 s	1.9	-0.1	-0.9	0.9
Day 4, 6 h, 5 s	1.9	0.2	0.2	0.9
Day 4, 6 h, 15 s	1.8	0.1	0.2	1
Day 4, 6 h, 30 s	2	0.2	0.3	0.9
(Mean of Day 4)	1.9	0.0	0.0	0.9
Mean of total	1.5	-0.1	-0.3	0.7

Note: the differences are calculated by the indicators of "fitting" minus the corresponding indicators of "fixed".

In the errors of both GNSS observation and geoid model, there is linear component and non-linear part. The former has linear relationship with the location, for example: the tilt of the whole network. The later is the systematic errors which have no linear relationship with the location, for example, internal errors in the GNSS-determined network. Equation (2) reveals that method of fitting free network to the known one only vertically move the GNSS determined network and tilt it with its geometrical centre as origin. Thus, it absorbs the linear part and the systematic part of the errors of GPS measurement and the geoid model, but it cannot eliminate the non-linear part. While the "fixed" method, which directly constrain the network with least square adjustment to the known one, is able to eliminate both the linear and the non-linear part of the error of GNSS measurement and the geoid model by changing the shape of the GNSS- determined network itself. Another potential reason of the practical advantage of "fixed" method is that "fixed" method keeps the degree of freedom because less common points is needed in the “fixed” method than in the “fitting” method. 6 common points is needed in the “fitting” method to solve the transformation parameters in Equation 2 and have 1 degree of freedom for each parameter. However, in the “fixed” method, 3 common points are needed to get degree of freedom of 2. Therefore, the “fixed” method is better when the number of common points for inclined plane transformation is so limited that the degree of freedom is low.

However, this comparison did not prove the “fixed” method is a better alternative. The least square adjustment used in the “fixed” method is designed to eliminate normal distributed, independent random errors instead of systematic errors (Wolf & Ghilani, 2006). However, in this application, it is used to remove all the errors in GPS observation including possible systematic errors. So, the result might be deformed. Moreover, the accuracy of the “fixed” method is easier to be effected by the accuracy of known points and their geometrical distribution because the GNSS-determined network will be changed to fit the known sites. Thus, it cannot be concluded that constrain the GNSS determined network during baselines adjustment is better than free network fitting. It can be applied if the accuracy of the GPS measurement, the geoid model and the known heights were good and the number of common points for inclined plane transformation was so limited that the degree of freedom was low.

Splitting the network into several sessions and degree of freedom

Practically in some occasions, the number of GNSS receivers and antennas available are limited. If so, the whole network cannot be measured in one session and therefore needed to be split into several sessions. In this study, such situations are simulated by measuring 9 sites with 6 receivers using L1/Lc (strategy of “9s6r” and “9s6rc”) and measuring 11 sites with 6 receives (“11s6r”) using L1, as introduced before in Chapter 4.1. They are compared in Table 15 with their comparable strategies, i.e., the strategies using the same settings except the number of receivers. See Lantmäteriet (1996) for how to plan and observe a network in several sessions.

Table 15. *Comparison between the spitted network and the original one*

Session Name	RMS of network fitting (RMSso, mm)	Local Network Error (mm)				
		RMS of height error (RMSco)	Standard deviation (Sco)	Max – min	Mean	RMS of standard error of unit weight (RMSso)
Leica antenna, L1 11 points, 6 receivers (11s6r)	7.8	5.5	4.1	14.3	3.8	2.8
Leica antenna, L1 9 points, 6 receivers (9s6r)	5.7	7.0	6.5	23.1	3.0	2.8
Leica antenna, L1 9 points, 6 receivers (9s6r_c)	2.6	3.8	2.7	9.9	2.8	4.3
Day 1, Leica antenna, L1, 11 points (D1_L1_1000)	2.8	4.3	1.0	3.1	4.2	2.5
Day 2, Leica antenna, L1, 11 points (D2_L1_1000)	1.9	5.5	0.8	2.1	5.5	2.4
Day 1, Leica antenna, Lc, 11 points (D1_Lc_1000)	2.6	4.1	1.4	3.6	3.9	3.1
Day 2, Leica antenna, Lc, 11 points (D2_Lc_1000)	2.2	5.2	0.9	2.8	5.1	3.9

The method measuring 11 sites with 6 receives (11s6r) achieves a good accuracy but with relatively poor repeatability (See Table 15): the RMS_{Co} using "11s6r" is found slightly worse by 1.2 mm than using "D1_L1_1000", but as good as using "D2_L1_1000". However, S_{Co} using "11s6r" is worse by 3.1 mm than using "D1_L1_1000" and by 3.3 mm than using "D2_L1_1000". The worse reliability is also indicated with the larger absolute values of maximum/minimum of C_0 and larger differences between them using "11s6r" (See Table 8 and Table 15). However, the mean of C_0 ($\overline{C_0}$) using "11s6r" is even better than using "D1_L1_1000" and "D2_L1_1000", showing that the extra error is random, rather than systematic. Therefore, it is thought possible to obtain good accuracy by measuring 11 sites with 6 receivers, but the repeatability is worse because of the low degree of freedom due to less redundancy of common points. The affection of degree of freedom will be analyzed later in this chapter. With declining of participating sites to 9, the accuracy and repeatability further decrease. The strategy "9s6r" yields worse result than "11s6r" by 1.5 mm in RMS_{Co}, by 2.4 mm in S_{Co}, by 0.8 mm in $\overline{Co}$ (see Table 15). Considering the S_{Co} and RMS_{Co} comprehensively, the final accuracy using "9s6r" might be larger than 1 cm (1σ). Therefore, this strategy is not recommended. The 9 sites included in this study has been selected so 6 sites are known points in the national height network, while 3 are sites in the local network. This is in order to keep the degree of freedom in the fit (inclined plane) to the national network at an acceptable level (dof: 6-3=3), and at the same time have redundancy in the connection of the local network to the GPS-determined points using the 1 parameter vertical transformation (dof: 3-1=2).

In the first (inclined plane) transformation, in which the desired heights of the local network is corrected, the degree of freedom is important to estimate the unknowns in Equation (2) more accurately. According to Equation (2), this transformation needs at least 3 common points to solve the unknowns. The redundancies enable averaging out the affection of a single point. With less redundancy, the estimates of those coefficients are more easily affected by error of a single point. Such misestimate of coefficients subsequently causes larger errors in one session and yields worse repeatability between different sessions (See Table 15). It is also shown in this comparison that the measurement with wavelength Lc is better when the degree of freedom is lower, see Table 15.

In conclusion, it is proved feasible to measure the network in several sessions and reach 1-cm accuracy although it is not as accurate and precise as measuring in one session. It is important to keep more degree of freedom to have better accuracy and precession, especially when the network cannot be measured in one session. When the degree of freedom is low, it is worth considering constraining the GPS determined network directly to the known one with least square adjustment, i.e., using the so-called “fixed” method, instead of the free network transformation. See Chapter 5.3.4. Unfortunately, it is difficult to specify which kind of split is acceptable or how to “safely” split the network with the limitation of data and purpose in this study. It is necessary to have serious consideration and evaluation when the network has to be split in several sessions.

The selection of common points

As introduced in Chapter 3 and Chapter 5.3.4, GPS-obtained network is firstly treated as a free network and subsequently fitted to the known heights of RH 2000. Common points between the two networks are needed to estimate translation value and rotation angles in Equation (2). Their accuracy effects the accuracy of the GPS-determined heights of the local network.

In this study, some benchmarks of RH 2000 included in the GPS measurement are used as common points. They are points of “1000-series” in most strategies. However, a trial with points of “2000-series” as common points is performed and compared to its comparable strategy to study possible errors due to inaccuracy of the known points (see Table 16).

Table 16. *The differences between applying different common points*

Session Name	Diff. RMS of Network fitting (ΔRMSso, mm)	Diff. local height error (mm)				
		ΔRMSco	ΔSco	Δ(Max-Min)	ΔMean	ΔRMSso
Leica antenna, Day 1, L1, 11	-3	1.6	-0.3	-1.1	1.8	0.1
Leica antenna, Day 2, L1, 11	-3.9	0.8	-0.7	-2.4	1	0.1

Note: the differences are calculated by the indicators using points of 1000-series minus the corresponding indicators using points of 2000-series

According to Table 16, the measurement using 2000-series point is found slightly better, and the height deviation due to the selection of known points is mostly from 0.1 mm to 1.9 mm. It can be seen that the selection of common points, especially the fixed point of the free network do affect the result because the absolute heights of all the other points are calculated from it. In this study, both points of 1000 and 2000-series are sufficient as known points.

5.4 Analysis on Possible Error Sources

According to the progress of this methodology and analysis above, possible existing errors in each step are generalized below in Table 17 and their possible sources are generalized in Table 18. Errors of GPS measurement and the geoid model are thought as the major errors exist in this study. The former includes possible errors of ordinary GNSS measurement, as well as the error due to the unsuitable antenna model of non-Javad antennas in this study. Among them, the affections of ionospheric effects and shifts in the satellite orbits are the most significant. The ionospheric effects can be significantly eliminated by using Lc while the shifts in the satellite orbits is tried to be reduced by using IGS final orbits (See Table 18).

Table 17*. Possible errors in each step*

Step	Possible errors
GPS Measurement, baseline processing and free network adjustment	Errors of GPS measurement
Coordinates transformation and normal heights calculation	Errors of GPS measurement
	Errors of the Geoid model
Network constraint	Radom error and the non-linear part of GPS measurement
	Radom and non-linear part of the geoid model

Table 18. Possible error sources

Errors	Error Sources	Solution*
Errors of GPS measurement	Errors in Antenna Models	Use the exact antenna model, or use the same type of antenna in one session
	Ionospheric effects	Using Lc
	Shifts in the satellite orbits	Using IGS final orbits. Include one known site.
	Clock errors of the satellites' clocks	Double differencing
	Multipath effect	How to select points!
	Tropospheric effects	Long observation time
	Others	
Errors of the Geoid model		
Other possible errors	Errors of the known points	Try with another set of known points

*Note: All the 3 errors may contain linear and non-linear component. Gross errors are checked and eliminated in each step; the linear part of systematic errors is mostly eliminated in the first (inclined plane) transformation.

The linear part of all sorts of errors are theoretically eliminated with the 1-dimentional 3-parameter transformation from the free network to the known one. After this transformation, the following errors remain: firstly, non-linear part of errors. They are theoretically incapable being eliminated by such transformation and therefore remain and contribute to the final errors. This non-linear part of errors includes non-linear part of errors in GPS measurement and geoid model. Many errors from various sources are suspected to be potentially non-linear systematic based on relevant studies (Colombo, 2000; Hofmann-Wellenhof, et el., 2001 etc.), see Table 19. However, due to the limitation of the topic of this study, only some of them are proved to be visible in this study. As discussed in Chapter 4.3.1., the existence of such non-linear systematic errors cause bias in the results and statistics.

Table 19. *Possible sources of systematic errors*

Error Sources	Potentially non-linear systematic*
Errors in Antenna Models	Yes (in this study)
Ionospheric effects	Yes
Shifts in the satellite orbits	Inconclusive
Clock errors of the satellites' clocks	No
Multipath effect	Inconclusive
Tropospheric effects	Yes
Errors of the Geoid model	Yes
Errors of the known points	No

*Note: Due to the limitation of the topic of this study, some error sources cannot be studied in detail. Refer to specific studies on such study for more information.

6 Conclusion and Discussion

6.1 Conclusion

Based on the field experiment of GPS measurement in Uppsala, this study evaluated the feasibility, accuracy and the effecting elements of connecting a local levelling network to RH 2000 with GNSS using SWEN 08 as geoid model. Essentially, this is determination of normal heights using GNSS. Different strategies of observations and computation are evaluated to investigate the effecting elements, with the aim to propose optimum strategies in different conditions for future practical applications. Some conclusions from this study are given below.

It is proved feasible to connect a local levelling network to RH 2000 with GPS, i.e., it is possible to obtain accurate normal heights with static carrier phase GPS observation and SWEN 08 as geoid model. The accuracy of most strategies tested fulfil the 1-cm level criterion (Chapter 5.2).

The GPS observation is proved sufficient with modern commercial standard antennas of good quality, like the Leica AX1202 GG, and dual frequency GPS receivers. However, the antennas should be well calibrated and the exact antenna phase centre variation (PCV) model must be used. Due to different manufacturers, possible deviations on the phase centre might occur due to the variations of same type of antenna (see Chapter 5.3.3). In practical work, it might be difficult to find the exact PCV antenna model sometimes. Since the phase centre cannot be measured exactly and it is different for L1 and L2, it is better to use the same type of antennas in a session. In such case, the systematic error can be considerably reduced even if an inaccurate antenna PCV model is applied.

It is shown in this study that session duration of 2 or 3 hours is sufficient for the studied application. Longer observation time generally yields better result. However, it is not an efficient option considering the accuracy and repeatability will not increase constantly and linearly with the increase of observation time (Chapter 5.3.1).

The result using ionospheric-free combination Lc is found as good as using L1, or even better sometimes in this study, but L1 is proposed in the former studies for smaller network. Considering the properties of the affection of ionosphere, L1 is proposed for baselines shorter than approximate 5 km and Lc is thought ideal for baselines longer than approximate 5 km (Chapter 5.3.2).

The “fixed” method, i.e., directly constrain the GNSS obtained network to the national network in the baseline adjustment, is found practically better than free network fitting in this study because it absorb both the linear and non-linear portion of errors in GPS measurement and the geoid model. However, theoretically, it is unstable because one of the prerequisites of least square adjustment is to exclude gross and systematic errors. Thus, the “fixed” method might be affected by systematic errors. Moreover, it is more seriously depend on the quality and distribution of known points in the national system than free network fitting because the shape of the GPS-obtained network will be changed to fit the known coordinates/ heights. However, it is thought to be a better alternative if there were not so many common points in the first (inclined plane) transformation, because it keeps the degree of freedom (Chapter 5.3.4). A practical procedure may be to first perform the free network adjustment and inclined plane transformation in order to check for gross errors. When this have been checked, perform the network adjustment using fixed heights at known points.

It is revealed in this study that the accuracy and repeatability decrease when the network is split and measured in several sessions and when the degree of freedom is low, due to less common sites. With only 3 trials, no “threshold” can be proposed to judge which kind of split is acceptable and how many common points is needed at least. However, it is suggested that such split should be considered and evaluated seriously in application when it is necessary. As to the degree of freedom, at least 3 common points are needed to solve Equation (2) and the necessary degree of freedom is determined by the quality (mainly the magnitude of non-linear errors within the GNSS determined network) of the GNSS measurement. Otherwise, constrain the GNSS obtained network directly to the national network, i.e., the “fixed” method, is thought theoretically better when the degree of freedom is low (Chapter 5.2.4 and 5.2.5).

At last, it is proved that the accuracy of the fixed point in the free network and the known sites in the national network will effect the accuracy of the result. Thus, it is important to apply accurate sites as fixed point and known points (Chapter 5.3.6 for details).

6.2 Discussion

The feasibility of GPS normal height obtainment with SWEN 08 is proved and effecting elements are analyzed in this study. Thus, the predetermined objective is accomplished. However, some limitations found should be discussed; some new problems and uncertainties arose need to be discussed and require further investigation.

The uncertainties of using Lc

In the comparison performed in Chapter 5.3.2 between L1 and Lc, Lc cannot be proved better although it yields better results in this study. The choice of wavelength mainly lies on two aspects: the length of baselines and the magnitude of both noise and ionospheric disturbances, which is uncertain. As to the ionosphere, it dramatically varies with "the main source of ionization" (Colombo, 2000): the sun. The solar cycle, other events such as travelling ionospheric disturbances (TID), in general geomagnetic and "space weather" conditions affect tremendously (Colombo, 2000). Considering the magnitude of the ionospheric affection varies, uncertainties would always exist in the practical result using Lc (Colombo, 2000). In spite of such uncertainties, in this study, it is showed that the result using Lc is as good as the result using L1, sometimes even better. Such uncertainty of using Lc is a subject worth further study.

The limitation of this study and overview on this subject

Firstly, although possible error sources are generalised in Chapter 5.4, they are not analyzed into detail. It is believed that there are still possible errors that remain unclear. For example, the two possible errors due to errors in horizontal domain mentioned in Chapter 3.2 cannot be proved and quantified in this study due to the limitation on time and subject of this study.

Secondly, the respective magnitude and propagations of each error and error source, i.e., their contribution to the overall error, remain unclear. Among the error sources listed in Table 18, only the errors due to antenna model can be quantified (see Figure 15). In the further study, more comparable tests that test each element separately while others isolated are required to provide a larger sample size. For some error sources that changing with some other elements, for example atmospheric affection, different set of observation in different circumstances should be performed to study how it changes. Eventually, each error source and the error propagation could be quantified. Such estimation would help to optimize strategy of observation in the future application. In this study, the result fulfils the criterion of 1-cm accuracy. However, with limitation on measurements available and topic of this study, the components of the error and the error in other environments remain unclear and need to be further analyzed.

Thirdly, among errors in this study, the error due to antenna model is the most significant in the measurement of Day 3 and Day 4. It tremendously distorted the results, caused uncertainties about the repeatability of this method and the practical performance when a mix between Ashtech and Javad "copies" of the AOAD/M_T antennas were used. Although an additional comparison is performed to clarify such uncertainties (See Chapter 5.3.3 and Figure 15), the actual accuracy in Day 3 and Day 4 cannot be revealed in this study. Therefore, it is suggested in Chapter 5.3.3 that the exact antenna model must be applied. It is also recommended to use the same type of antennas in a session. In this case, the systematic error can be easily eliminated even if antenna model applied is not accurate. In future studies based on the same data set, antenna model should be improved to compute the actual accuracy of the measurement in Day 3 and Day 4.

Fourthly, in this study, some tendencies are found, but lack of evidence to draw a conclusion. In Table 20, the uncertainties remained in this study are generalized and the possible solutions in further study are proposed. The first four items in Table 20 are found practically exist but cannot be proved with limited data and experiments in this study. Moreover, the purpose of this study is to test this methodology and its affecting elements, therefore some detailed study can not be included. More detailed further study required in those specified fields to further optimize this method.

Table 20. *Uncertainties remained in this study*

Uncertainties	Further studies required
The pros and cons of Lc and how it changes with other elements (length of baseline, time etc.).	1. Quantitative study with more data 2. Professional research on ionosphere and its affection to GNSS measurement
If there is any atmospheric affection in the differences using different antennas? If it exist, how much of it?	1. Professional research on atmospheric (ionospheric, tropospheric etc.) affection to GNSS measurement
The quantitative affection of splitting the network into sub-sessions	1. Quantitative study with more data and mathematical modelling and analysis
The quantitative affection of less degree of freedom	1. Mathematical modelling and analysis based on the algorithm of network fitting
The quantitative affection of methods of network fitting	1. Mathematical analysis on different methods of network fitting 2. Study showing the specified Conditions applying the "fixed" method if it can be used
Error sources of this methodology, their quantitative affection propagation	1. Quantitative study with more data

At least, in this study, only GPS signal is used. That is because some receivers used in the GPS field observation campaign were unable to track GLONASS signals. However, due to their theoretic similarity, the general conclusion obtained in this study is applicable to other systems. Other systems, including GLONASS signal can be tested in the further study.

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References

Becker, J-M., 1985, The Swedish experience with motorized levelling new techniques and tests, The NAVD Symposium, April pp. 21- 26, 1985; Lantmäteriet report number: LMV-RAPPORT 1985:7

Becker, M., Zerbini, S., Baker, T., Bürki, B., Galanis, J., Garate, J., Georgiev, I., Kahle, H.-G., Kotzev V., Lobazov, V., Marson, I., Negusini, M., Richter, B., Veis, G. and Yuzefovich, P., 2002. Assessment of height variation by GPS at Mediterranean and Black Sea coast tide gauges from the SELF projects, Global and Planetary Change 34(2002) 5-3

Benahmed Daho, S.A., Kahlouche, S., and Fairhead, J.D., 2006. A Procedure for Modelling the Differences Between the Gravimetric Geoid Model and GPS/levelling Data with an Example in the North Part of Algeria, Computers & Geosciences 32 (2006) 1733–1745

Colombo, O.L., 2000. Ionospheric Tomography Helps Resolve GPS Ambiguities on the Fly At Distance of Hundreds of Kilometers During Increased Geomagnetic Activity, IEEE "PLANS 2000" Meeting, San Diego, 13-16 March 2000

Engelis, T., Rapp, R.H. and Tscherning, C.C., 1984. The precise computation of geoid undulation differences with comparison to results obtained from the global positioning system, Geophysical Research Letters, vol. 11, no. 9, pp. 821-824.

Engelis, T., Rapp, R.H. and Bock, Y., 1985. Measuring orthometric height differences with GPS and gravity data, Manuscripta Geodaetica, vol. 10, no. 3, pp. 187-194

Eriksson, P., 2009. Anslutning av lokala höjdnät till RH 2000 Stomnätsmätning med statisk GNSS-mätning, Projektrapport – ISSN 280-5731 Lantmäteriet (in Swedish)

Featherstone, W.E., 2008. GNSS-based Heighting in Australia: Current, Emerging and Future Issues, Spatial Science, vol. 53, no. 2, December 2008

Hofmann-Wellenhof, B., Lichtenegger, H. and Collins, J., 2001. GPS theory and practice (Fifth, revised edition), Springer-Verlag Wien New York

Hu, W., Sha, Y., and Kuang, S., 2004. New Method for Transforming Global Positioning System Height into Normal Height Based on Neural Network, Journal of Surving Engineering, February 2004, DOI: 10.1061/(ASCE) 0733-9453(2004) 130:1 (36)

Lantmäteriet 1996. Handbok till mätningskungörelsen Geodesi, GPS (Andra utgåvan), HMK-Ge: GPS, ISBN 91-7774-061-0 (in Swedish). http://www.lantmateriet.se/hmk , cited 03/2011

Lantmäteriet 2009a. Height systems - RH 2000, http://www.lantmateriet.se/templates/LMV_Page.aspx?id=4211, cited 04/2010

Lantmäteriet 2009b. The geoid - Swedish geoid models, http://www.lantmateriet.se/templates/LMV_Page.aspx?id=11011, cited 04/2010

Lantmäteriet 2009c. The user manue of Gtrans 3.63 (in Swedish)

Lilje, M., 2006. The New Swedish Height System RH 2000 and Geoid Model SWEN 05LR, XXIII FIG Congress, Munich, 8-13 October 2006

Mårtensson, S-G., 2002. Height Determination by GPS – Accuracy with Respect to Different Geoid Models in Sweden, FIG XXII International Congress, Washington, 19-26 April 2002

Trimble Navigation Ltd., 2002. Trimble Total Control™ User Guide

Wlof, P.R. and Ghilani, C.D., 2006. Elementary Surveying (an introduction to Geomatics), Pearson Education, Inc. pp. 396 – 410

Yang, Z. and Chen, Y., 1999. Determination of Local Geoid with Geometric Method: Case Study, Journal of Surveying Engineering, August 1999.

Zhang, Q. and Li, J., 2005. The principle and application of GPS survey (in Chinese), Science Press of China

Zilkoski, D.B., 1990. Establishing Vertical Control Using GPS Satellite Surveys, FIG June 1990

Ågren, J., 2009. Beskrivning av de nationella geoidmodellerna SWEN08_RH2000 och SWEN08_RH70 (in Swedish), Rapportserie: Geodesi och Geograkiska informationssystem LMV-Rapport 2009:1

Appendix

1. Installing the antenna models

As mentioned in Chapter 3.1, in this study, the phase centre variation (PCV) models of Leica AX1202GG and Javad JNSCR_C146-22-1 antennas are not included in TTC. Therefore, they must be manually installed by modifying the file "antenna.ini" (Trimble Navigation Ltd., 2002). The modified file used in this study is included below.

Antenna.ini

```
[AntDatabaseInfo]
Version=6.70
Language=SWE

; Instructions
;-------------------------------------------------------------------------
; - Use a unique [key] for each new antenna entry.
;    (If you have serial number dependent antenna calibrations this
[key]
;   must be a combination of antenna name and serial number.)
; - The Type number must be larger than 10000 and unique.
; - The following items must be unique:
;     [key]
;     Name
;     DCName              (not needed)
;     CharCode            (not needed)
;     Type

[PhaseCorrTables]
Set1=Trimble,PhaseCorrTable,"Default Trimble Calibration"
Set2=NGS,NGSCorrTable,"US National Geodetic Survey,
ant_info.003"
Active=Set2

[AntennaGroup]
Group1=Custom
Group2=Survey

[Custom]
Name=Custom
ReadOnly=0

[Survey]
Name=Survey
ReadOnly=0
```

```
Ant1=JNSCR_C146-22-1,0,default=0

[JNSCR_C146-22-1]
Name=JNSCR_C146-22-1
Manufacturer=Javad Positioning Systems
Class=Survey
PartNumber=
CharCode=
Type=10001
MeasMethod0=0.00000,0.11000,0.00000,"Undersida av antennfästet"
RINEXMethod=0
RINEXName=JNSCR_C146-22-1
RINEXName=JNSCR_C146-22-1 NONE
PhaseCorrTable=
NGSCorrTable=jns_c146.ngs
Freq=2
GraphicsFile=jns_c146.jpg
```

```
copy E:\Thesis_work\GPS_data\Uppsala_2008a_februari\Dygn_1_2008-02-18--19\Tim_03\Sess-DEF\*.* D:\new\*.*
```

2. Routine of the Excel VBA macro used for statistics

Due to the complexity of file arrangement and Excel operation, an Excel VBA macro is developed to automate some procedures of statistics (see Chapter 4.3.2). The routine and subs of this macro is listed below.

```
Private Function FileNames() As Variant
Dim Filter As Variant
Dim DefaultFilter As Integer
Filter = "Text Files (*.txt),*.txt," & "Gtrans Result Files (*.gp),*.gp"
DefaultFilter = 2
'Filter and default settings
' Default Disk and path
ChDrive ("d")
ChDir ("d:\coordinates_results")
'GetOpenFilename
FileNames = Application.GetOpenFilename(Filter, DefaultFilter,
"Open Gtrans result files (Translation)...", , True)
End Function

Private Function ReadFile(filenum As Integer) As Variant 'Read the
content of the file into an array
Dim col As Integer
Dim content(300) As String
col = 1
Do While Not EOF(filenum)
  Line Input #1, content(col)
  col = col + 1
 Loop
 ReadFile = content
End Function

Sub RangeWorkSheet()
    Columns("A:A").ColumnWidth = 36.5 'adjust width and add
header
    Range("A1") = "File Name"
    Range("B1") = "So"
    Range("D1") = "Co"
    Range("E1") = "Session Length"

    Columns("A:E").Select
```

```
   Selection.Sort Key1:=Range("E2"), Order1:=xlAscending,
Key2:=Range("A2") _
       , Order2:=xlAscending, Header:=xlYes, OrderCustom:=1,
MatchCase:=False _
       , Orientation:=xlTopToBottom, SortMethod:=xlPinYin,
DataOption1:= _
       xlSortNormal, DataOption2:=xlSortNormal

End Sub
Sub FillIn(content As Variant, col2 As Integer, fname As String)
    Dim filetype As String
    filetype = Right$(LCase$(fname), 5)
    ActiveCell.Offset(col2) = Trim(fname)
    ActiveCell.Offset(col2, 1) = "0" & Mid(Trim(content(28)), 59, 100)
    If filetype = "-c.gp" Then
      ActiveCell.Offset(col2, 2) = "Co ="
      ActiveCell.Offset(col2, 3) = "0" & Mid(Trim(content(33)), 23, 12)
      If Mid(Trim(content(33)), 22, 1) = "-" Then ActiveCell.Offset(col2,
3) = ActiveCell.Offset(col2, 3) * -1
    End If
    ActiveCell.Offset(col2, 4) = InStr(fname, "_") - 3 'count probable
session lenght
End Sub
Sub Compute() 'Compute the current worksheet only
Dim SessionNum As String
Dim i As Integer, RangeStart As Integer, NumRows As Long
Dim CalRange As Range
RangeStart = 2
NumRows = ActiveSheet.UsedRange.Row - 1 +
ActiveSheet.UsedRange.Rows.Count 'Poer, How to find the last row
that contain data in Excel, http://www.excelvbamacro.com,
consulted 11/11, 2009
   Range("F1") = "Session #"   'header
   Range("G1") = "RMS So"
   Range("H1") = "RMS Co"
   Range("I1") = "Standard Divation Co"
   Range("J1") = "Max Co"
   Range("K1") = "Min Co"
   Range("L1") = "Diff Co"
   Range("M1") = "Mean Co"
   Range("n1") = "Number of Sessions"
For i = 2 To NumRows
  ActiveWorkbook.ActiveSheet.Cells(i, 1).Activate
  SessionNum = Left(ActiveCell, 2)

  If SessionNum <> Left(ActiveCell.Offset(1, 0), 2) Or
ActiveCell.Offset(0, 4) <> ActiveCell.Offset(1, 4) Then
  Cells(i, 6) = SessionNum                          'fill in the session
length
```

```
  Set CalRange = ActiveSheet.Range(Cells(RangeStart, 2), Cells(i, 2))
'range selection
  Cells(i, 7).Formula = "=SQRT(SUMSQ(" &
CalRange.Columns(1).Address(False, False) & ")/COUNT(" &
CalRange.Columns(1).Address(False, False) & "))" 'RMS So
Calculation
  Cells(i, 14) = i - RangeStart + 1
   If Cells(2, 4) <> "" Then 'if Co exist, Calculate Standard divation,
Max, Min, Diff of Co
    Set CalRange = CalRange.Offset(0, 2)
    Cells(i, 8).Formula = "=SQRT(SumSq(" &
CalRange.Columns(1).Address(False, False) & ") / Count(" &
CalRange.Columns(1).Address(False, False) & "))"
    Cells(i, 9).Formula = "=StDev(" &
CalRange.Columns(1).Address(False, False) & ")"
    Cells(i, 10).Formula = "=Max(" &
CalRange.Columns(1).Address(False, False) & ")"
    Cells(i, 11).Formula = "=Min(" &
CalRange.Columns(1).Address(False, False) & ")"
    ActiveWorkbook.ActiveSheet.Cells(i, 10).Activate
    Cells(i, 12).Formula = ActiveCell - ActiveCell.Offset(0, 1)
    Cells(i, 13).Formula = "=average(" &
CalRange.Columns(1).Address(False, False) & ")"
   End If

  RangeStart = i + 1
  End If
Next i
End Sub
Sub ComputeAll() 'go through and compute all the worksheets
Dim i As Integer
For Each ws In ThisWorkbook.Worksheets
If ws.Name <> "Result" Then
ws.Activate
Call RangeWorkSheet
Call Compute
End If
Next
End Sub
Sub IfExist() 'Look for worksheet "Result", create if it's not exist, to
provide a place for the final result
Dim WorksheetResult As Worksheet
For Each ws In ThisWorkbook.Worksheets 'go through all the
worksheets
 If ws.Name = "Result" Then
 ws.Activate
 Exit Sub
 End If
```

```
Next
Set WorksheetResult = ThisWorkbook.Worksheets.Add
WorksheetResult.Name = "Result"
End Sub
Sub orgnize()
Dim i As Integer, ii As Integer, ComRange As Range
Call IfExist
'ThisWorkbook.Sheets("Result").range("a2").Select
With ThisWorkbook.Sheets("Result").Range("a2")
  .Offset(0, 0) = "Session Name"
  .Offset(0, 1) = "Day"
  .Offset(0, 2) = "Session Length"
  .Offset(0, 3) = "Number of Sessions"
  .Offset(0, 4) = "RMS of network fitting (mm)"
  .Offset(0, 5) = "RMS of height error"
  .Offset(0, 6) = "Standard deviation"
  .Offset(0, 7) = "Max"
  .Offset(0, 8) = "Min"
  .Offset(0, 9) = "Max - min"
  .Offset(0, 10) = "Mean"
  .Offset(0, 11) = "RMS of standard error of unit weight"
End With

For Each ws In ThisWorkbook.Worksheets
  If ws.Name <> "Result" And Right(ws.Name, 1) = "T" Then
    For ii = 2 To ws.UsedRange.Row - 1 + ws.UsedRange.Rows.Count
      If ws.Cells(ii, 6) <> "" Then
       With ThisWorkbook.Sheets("Result").Range("a3")
        .Offset(i, 0) = ws.Cells(ii, 6) '"Session Name"
        Select Case ws.Cells(ii, 6) '"Day"
          Case 11, 13, 15
           .Offset(i, 1) = "1"
          Case 12, 14, 16
           .Offset(i, 1) = "2"
          Case 21, 23
           .Offset(i, 1) = "3"
          Case 22, 24
           .Offset(i, 1) = "4"
          Case "9s", 10
           .Offset(i, 1) = "1+2"
        End Select
        .Offset(i, 2) = ws.Cells(ii, 5)  '"Session Length"
        .Offset(i, 3) = ws.Cells(ii, 14) '"Number of Sessions"

        .Offset(i, 5) = ws.Cells(ii, 8) * 1000 '"RMS of height error"
        .Offset(i, 6) = ws.Cells(ii, 9) * 1000 'Standard deviation
        .Offset(i, 7) = ws.Cells(ii, 10) * 1000 'Max
        .Offset(i, 8) = ws.Cells(ii, 11) * 1000 'Min
        .Offset(i, 9) = ws.Cells(ii, 12) * 1000 'Max-Min
```

```
        .Offset(i, 10) = ws.Cells(ii, 13) * 1000 'Mean
        .Offset(i, 11) = ws.Cells(ii, 7) * 1000 '"RMS of standard error of
unit weight"
        End With
        i = i + 1
      End If
    Next ii
  End If
Next

For Each ws In ThisWorkbook.Worksheets
 If ws.Name <> "Result" And Right(ws.Name, 1) <> "T" Then
  For i = 2 To ws.UsedRange.Row - 1 + ws.UsedRange.Rows.Count
    If ws.Cells(i, 6) <> "" Then
     For ii = 3 To Worksheets("Result").UsedRange.Row - 1 +
Worksheets("Result").UsedRange.Rows.Count
      If ws.Cells(i, 6) = Worksheets("Result").Cells(ii, 1) And ws.Cells(i,
5) = Worksheets("Result").Cells(ii, 3) Then
Worksheets("Result").Cells(ii, 5) = ws.Cells(i, 7) * 1000
     Next ii
    End If
  Next i
 End If
Next

'the following code format the "result" table and range the result

    Columns("B:B").ColumnWidth = 3.44
    Columns("C:C").ColumnWidth = 12
    Columns("D:D").ColumnWidth = 11.56
    Columns("E:E").ColumnWidth = 22
    Columns("F:F").ColumnWidth = 16
    Columns("G:G").ColumnWidth = 15.11
    Columns("H:H").ColumnWidth = 4.56
    Columns("I:I").ColumnWidth = 4.78
    Range("E3:L27").Select
    Range("E27").Activate
    Selection.NumberFormat = "0.0"
    Range("A2:L27").Select
    Selection.Sort Key1:=Range("A3"), Order1:=xlAscending,
Key2:=Range("B3") _
        , Order2:=xlAscending, Key3:=Range("C3"),
Order3:=xlAscending, Header:= _
        xlGuess, OrderCustom:=1, MatchCase:=False,
Orientation:=xlTopToBottom, _
        DataOption1:=xlSortNormal, DataOption2:=xlSortNormal,
DataOption3:= _
        xlSortNormal
```

```
End Sub
Sub main()
Dim FileName As Variant 'file names
Dim i As Integer, col2 As Integer, sessionlength As Integer, col3 As
Integer 'number of files, number of cols in worksheetInner,Session
Length
col2 = 1
col3 = 1
Dim content As Variant, nameArr As Variant, NeatFileName As
String  'file  content
Dim WorkSheetName As String
Dim WorkSheetOuter As Worksheet
Dim WorksheetInner As Worksheet
FileName = FileNames()
If Not IsArray(FileName) Then Exit Sub 'Exit on cancel

Set WorkSheetOuter = Worksheets.Add 'add 2 new worksheets for
outer loop and inner points fitting
Set WorksheetInner = Worksheets.Add

For i = LBound(FileName) To UBound(FileName) ' Open Files
  Open FileName(i) For Input Access Read As #1

  content = ReadFile(1) 'read files, the content has been read into an
array, and ready for filled into worksheets

  'If i = LBound(Filename) Then    'rename 2 worksheets
  nameArr = Split(FileName(i), "\")
  NeatFileName = nameArr(UBound(nameArr))
  'sessionlength = InStr(NeatFileName, "_") - 3 'count probable session
lenght
  WorkSheetName = Mid(Trim(NeatFileName), 1, 2)
  WorkSheetOuter.Name = WorkSheetName & "_L"
  WorksheetInner.Name = WorkSheetName & "_T"
  'End If

  Select Case Right$(LCase$(FileName(i)), 5)
    Case "-c.gp"
      WorksheetInner.Activate
      Call FillIn(content, col2, NeatFileName)
      col2 = col2 + 1
      Call RangeWorkSheet
    Case "00.gp"
      WorkSheetOuter.Activate
      Call FillIn(content, col3, NeatFileName)
      col3 = col3 + 1
      Call RangeWorkSheet
    Case Else
```

```
    MsgBox (FileName(i) & " is not a valid file.")
    End Select
Close #1
WorkSheetOuter.Activate
Call RangeWorkSheet
WorksheetInner.Activate
Call RangeWorkSheet
Next i

Set WorkSheetOuter = Nothing
Set WorksheetInner = Nothing

End Sub
```

Reports in Geodesy and Geographical Information Systems from Lantmäteriet (the National Land Survey of Sweden)

2007:11 Lilje Christina, Engfeldt Andreas, Jivall Lotti: Introduktion till GNSS.

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2009:5 Bosrup Susanna & Illerstam Jenny: Restfelshantering med Natural Neighbour och TRIAD vid byte av koordinatsystem i plan och höjd.

2010:1 Reit Bo-Gunnar: Om geodetiska transformationer (finns även på engelska med titeln *On geodetic transformations*).

2010:2 Odolinski Robert: Studie av noggrannhet och tidskorrelationer vid mätning med nätverks-RTK.

2010:3 Odolinski Robert: Checklista för nätverks-RTK.

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2010:6 Engfeldt Andreas & Odolinski Robert: Punktbestämning i RH 2000 – statisk GNSS-mätning mot SWEPOS.

2010:7 Lord Jonas: Test av GNSS-mottagare från DataGrid.

2010:11 Ågren Jonas & Engberg Lars E: Om behovet av nationell geodetisk infrastruktur och dess förvaltning i framtiden.

2011:2 Jansson Jakob: Undersökning av mätosäkerheten i det förtätade SWEPOS-nätet i Stockholmsområdet – vid mätning med nätverks-RTK.

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